REGINA PACIS COLLEGE GARKI ABUJA

SS3 LESSON NOTE

FURTHER MATHEMATICS

WEEK 1

PARTIAL FRACTION

1) $\frac{f(x)}{(x+a)(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+b)}$ ...f(x) = A(x+b) + B(x+a), then to find A set the Denominator equal to zero, the same thing applicable to B this method is call COVER UP METHOD

Example Resolve $\frac{3x+5}{2x^2+5x-3}$ $into\ partial\ fraction$

Factorized the denominator to get (x-3)(2x+1)

$\frac{3x+5}{2x^2+5x-3}$=$\frac{A}{(x-3)} + \frac{B}{(2x+1)}$

3x+5 = A(2x+1) + B(x-3)...........**, to find A set x-3=0, implies x=3

3(3)+5 = A(2X3+1) +B(3-3)

9+5 = A(6+1) + B(0)

14 = 7A

Divide both side by 7 to get A=2

To get B set 2x+1=0, 2x=-1, x=-(1/2)=-0.5, then substitute x into **

3(-0.5)+5 = A(2X-0.5 +1) + B(-0.5-3)

3.5 = -3.5B,

B=-1

$\frac{3x+5}{2x^2+5x-3}$=$\frac{2}{(x-3)} - \frac{1}{(2x+1)}$

Another Method

3x+5=2Ax+A+Bx-3B

3x+5=(2A+B)x+A-3B

By equating their coefficient

2A+B=3......................(1)

A-3B= 5....................(2)

2) $\frac{f(x)}{(x+a)^2(x+b)} = \frac{A}{(x+a)} + \frac{B}{(x+a)^2} + \frac{C}{(x+b)}$

3) $\frac{f(x)}{ax^2+bx+c)(x+d)} = \frac{Ax+B}{ax^2+bx+c} + \frac{C}{(x+d)}$

Another Example

$\frac{3x}{(x-1)(x+2)}$=$\frac{A}{(x-1)} + \frac{B}{(x+2)}$

3x=A(x+2)+B(x-1)

3x=Ax+2A+Bx-B

3x=(A+B)x+2A-B

A+B=3...........1

2A-B=0..........2

$$\frac{3x+1}{(x-1)^2(x+2)}=\frac{A}{(x-1)}+\frac{B}{(x-1)^2}+\frac{C}{(x+2)}$$

3x+1=A(x-1)(x+2)+B(x+2)+C(x-1)(x-1)

To find **B** set x-1=0, x=1

4=3B, B=4/3

To find C set x+2=0, x=-2

-5=9C

C=-5/9

To find A set x=0

1=-2A+2B+C

A=5/9

3x+1=A(x-1)(x+2)+B(x+2)+C(x-1)(x-1)

3x+1=A(x^2+x-2)+B(x+2)+C(x^2-2x+1)

3x+1=Ax^2+Ax-2A+Bx+2B+Cx^2-2Cx+C

3x+1=(A+C)x^2+(A+B-2C)x-2A+2B+C

By equating the coefficient

A+C=0..................1

A+B-2C=3...........2

2A+2B+C=1..........3

Resolve the following into partial fraction

1. $\frac{3-4x-x^2}{(x^2-4x+3)x}$
2. $\frac{2(6x+1)}{(4x^2-1)(2x+3)}$
3. $\frac{5x^2+17x+13}{(x+2)^2(x+1)}$

WEEK 2
TOPIC: INTEGRATION
Integration: This is defined as anti- differentiation. Suppose, $y = x^3 + 2x$, the first derivative is $3x^2 + 2$. ($dy/dx = 3x^2 + 2$)
then the anti – derivative of $3x^2 + 2 = x^3 + 2x$
Thus, integration is the reverse process of differentiation and denoted by the symbol ∫.
If $dy/dx = x^n$, then $\int dy/dx = \frac{x^{n+1}}{n+1} + C \quad (n \neq -1)$
where c is the arbitrary constant.

INDEFINITE INTEGRAL CONCEPTS
General Concept
Example: Evaluate the following integrals:
1. $\int x^2\, dx$ 2. $\int x^{5/2}\, dx$ 3. $\int 4/ x5\, dx$ 4. $\int \sqrt{x^8}$ 5. $\int (7x^4 + 2)\, dx$ 6. $\int (x^5 + 2x^4 – x^3 + 6)\, dx$

Solution;
1. $\frac{x^3}{3} + C$ 2. $\frac{x^{5/2+1}}{5/2+1} = \frac{2x^{7/2}}{7} + c$ 3. $\int 4x^{-5}\, dx = \frac{4x^{-5+1}}{-5+1} = -4^{x-4} + C$
4. $\int x^4 = x^{4+1} = x^5 + C$ 5. $\frac{7x^{4+1}}{4+1} + 2x^{0+1} = \frac{7x^5}{5} + 2x + C$
6. $\frac{x^6}{6} + \frac{2x^5}{5} – \frac{x^4}{4} + 6x + C$

NB: Integral of a constant is not zero but the variable in the question.

Evaluation: Evaluate the following integrals; 1. $\int (12x^3 – x^6 + 1/x^2)\, dx$ 2. $\int x^2(3x^2 + 4x)\, dx$

Trigonometric integral:
The trigonometric integrals can be summarized in the following table. Remember that this is the reverse process of differentiation.

F(x)	∫ f(x)dx
Sin x	- cos x + c
Cos x	sin x + c
Sec^2x	tan x + c
$Cosec^2x$	- cot x + c
Sec x tan x	sec x + c
e^x	$e^x + c$
1/x	ln x + c

Example: Evaluate each of the following integrals.
1. ∫ sin x – 5 cos x)dx 2. $\int (5\sin x + 3x^2)dx$
Solution:
1. . ∫ sin x – 5 cos x)dx = ∫sin x dx - ∫5 cos x dx
= -cos x – 5 (sin x) + c
= - cos x – 5sin x + c
2. $\int (5\sin x + 3x^2)dx = \int 5 \sin x\, dx + \int 3x2\, dx$
$= -5\cos x + x^3 + c$
3. $\int e^{2x}\, dx = e^{2x}/2 + c$

Evaluation: Evaluate the integrals:
1. ∫ (3 cos x + 2 sin x) dx 2. $\int e^{2x2 + 5x}\, dx$

INTEGRATION BY ALGEBRAIC SUBSTITUTION

Sometimes integral are not given in the standard form, such integral are then reduced to standard form format before evaluation by algebraic substitution.

Suppose, an integral is given in the form $\int$ **f(ax + b)^n dx**

Then, the algebraic substitution is to represent the function in the bracket by any letter.

Let $\quad$ **u = (ax + b) $\quad$ du/dx = a, $\quad$ dx = du/ a**

$$\int u^n\, dx = \int u^n\, du/a$$

$$= 1/a \int u^n\, du$$

Example:

Evaluate the following integrals.

1. $\int (2x^2 - 5x)^4\, dx$ $\quad$ 2. $\int \frac{7}{(5x-4)^5}\, dx$ $\quad$ 3. $\int x\cos 2x^2\, dx$ $\quad$ 4. $\int x^2 \sqrt{(x^3+5)}dx$

Solution:

1. $\int (2x^2 - 5x)^4\, dx$ $\quad$ let $u = 2x^2 - 5x$, $du/dx = 4x - 5$, $dx = du/4x$ **-5**

$\int u\ 4\ du/\ 4x -5 = \frac{u^5}{5(4x-5)} + c$

$= \frac{(2x^2 - 5x)^5}{20x - 25} + c$

2. $\int \frac{7}{(5x-4)^5}\, dx$ $\quad$ let $u = (5x - 4)$, $du/dx = 5$, $dx = du/5$

$\int 7\, u^{-5}\, du/5 \quad = \frac{7u^{-4}}{5x - 4} + c$

$= \frac{7(5x-4)^{-4}}{-20}$

3. $\int x \cos 2x^2\, dx$ $\quad$ let $u = 2x^2$, $du/dx = 4x$, $dx = du/4x$

then; $\int x \cos 2x^2\, dx = \quad \int x \cos u\, du/4x = \frac{1\ x\ (\sin u)}{4} = \frac{1 \sin 2x^2}{4} + c$

4. $\int x^2 \sqrt{(x^3+5)}dx$ $\quad$ let $u = x^3 + 5$, $du/dx = 3x^2$, $dx = du/3x^2$

$\int x^2 \sqrt{(x^3+5)}dx \quad = \int x^2 u^{1/2}\, du/3x^2 = \frac{1\ x\ u^{3/2}}{3/2\ x\ 3} = \frac{2\ (x^3+5)^{3/2}}{9} + c$

Evaluation:

Evaluate the following integrals:

1. $\int(5x - 7)^{7/2}dx$ $\quad$ 2. $\int\cos 9x\, dx$ $\quad$ 3. $\int x\cos 2x\, dx$.

INTEGRATION BY PARTS

This technique is uniquely useful in evaluating integrals that are not in the standard form. Such integrals cannot be solved by algebraic substitution.

From the product rule of differentiation, it can be generalized thus;

$$\int v\,du = uv - \int u\,dv.$$

Example: Evaluate the following integral by parts.

1. $\int 2x \sin x\, dx$ $\quad$ 2. $\int e^{2x}\cos 2x\, dx$

solution:

1. $\int 2x \sin x\, dx$ $\quad$, let $v = 2x$, $dv/dx = 2$, $dv = 2dx$

$\int du = \sin x\, dx$

$u = -\cos x$

$\int v\,du = uv - \int u\,dv.$

$\int x^2 \sin x = - x^2\cos x - \int- \cos x\ x\ 2\, dx$

$= - x^2\cos x + 2\int \cos x$

$= - x^2\cos x + 2 \sin x + c$

2. $\int x^2e^x\ dx$, let $v = x^2$, $dv/dx = 2x$, $dv = 2xdx$
$du = e^x$ $u = e^x\ dx$
$\int vdu = uv - \int udv$

$$\int x^2e^x = e^x x^2 - \int e^x\ 2xdx$$
$$= x^2 e^x - 2\int e^x\ x\ dx$$

the integral part in the RHS will have to be evaluated using integration by parts;
thus, $v = x$, $dv/dx = 1$, $dv = dx$, $du = e^x$, $u = e^x$
$\int vdu = uv - \int udv$

$$\int x\ e^x = e^x . x - \int e^x dx$$
$$= e^x.x - e^x$$

finally, $\int x^2e^x = x^2\ e^x - 2(e^x.x - e^x)$
$$= x^2e^x - 2xe^x + 2e^x + c$$

Evaluation:
Evaluate 1. $\int x^2\cos x\ dx$ 2. $\int x^3\ e^{-x}\ dx$

INTEGRATION BY PARTIAL FRACTION
Sometimes rational functions are not expressed in the proper standard form; such function can be evaluated by transforming them into standard form through partial fractions. The knowledge of partial fractions is needed here to evaluate the functions.

Example; Integrate each of the following with respect to x;
1 $\dfrac{2x + 3}{(2x + 1)(x - 1)}$ 2. $\dfrac{x + 8}{(x^2 + 3x + 2)}$
solution:
1. resolve into partial fraction; $\dfrac{2x + 3}{(2x + 1)(x - 1)} = \dfrac{A}{(2x + 1)} + \dfrac{B}{(x - 1)}$
$$2x + 3 = A(x-1) + B(2x + 1)$$
when $x = 1$, $2(1) + 3 = B(2 + 1)$
$5 = 3B$, $B= 5/3$
when $x = -1/2$, $2(-1/2) + 3 = A(-1/2 - 1)$
$2 = - 3/2\ A$ $A = - 4/ 3$

$$;\ \mathbf{\frac{2x + 3}{(2x + 1)(x - 1)} = \int \frac{-4}{3(2x + 1)} + \int \frac{5}{3(x - 1)}dx}$$
$$= \frac{-4\ \ln (2x + 1)}{2\ x\ \ 3} + \frac{5\ \ln (x - 1)}{3}$$
$$= - 2/3 \ln (2x + 1) + 5/3 \ln (x - 1) + c$$

2. $\dfrac{x + 8}{(x^2 + 3x + 2)} = \dfrac{x + 8}{(x + 1)(x + 2)} = \dfrac{A}{x + 1} + \dfrac{B}{x + 2}$
$$x + 8 = A(x+2) + B(x+1)$$
when $x = -2$,
$- 2 + 8 = B(-2+1)$
$6 = -B$, $B = - 6$
when, $x = -1$, $- 1 + 8 = A(-1 + 2)$
$7 = A$.

thus, $x + 8 = \int \dfrac{7}{x + 1} + \int \dfrac{- 6}{x + 2}$
$$= 7\ln (x+ 1) - 6 \ln(x+2) + c$$

Evaluation

Integrate by partial fraction.

1. $\dfrac{4x + 3}{(x - 3)(x+2)}$ 2. $\dfrac{1}{(x^2 + 3x + 2)}$

GENERAL EVALUATION/REVISIONAL QUESTIONS

1.Find the derivatives of the following with respect to x;

(a) $y = (15 + 5x)(1 + 2x)$ (b) $y = (1 + 2x)^{12}$ (c) $y = 3x^2 (3 - 2x + 4x^2)^{½}$

2. Given that the gradient function of a curve is $8x - 2$, find the equation of the curve at point (2, 4)
3. Find $\int(x^2 + 1)(x^3 - 2)dx$
4. Find $\int x^2 e^{2x}dx$

Reading Assignment: Read Integration, Page 31 – 46 Further Mathematics project III.

WEEKEND ASSINGMENT

1. Evaluate $\int (x^5 + 3)dx$. A. $x^6/6 + 3x + c$ B. $x^5/6 + c$ C. $x^6/6 + c$
2. Evaluate $\int \cos 7x\, dx$ A. $7\sin 7x$ B. $1/7 \sin 7x + c$ C. $7\sin 7x + c$
3. Integrate the function; $(3x + 5)^5$ wrt x .A $12(2x+3)^6 + c$ B. $\dfrac{(2x +3)^6}{12} + c$ C. $\dfrac{(3x + 5)^6}{18} + c$
4. Find $\int(x+1)(x^2 - 2)dx$ A. $\dfrac{x^4}{4} + \dfrac{x^3}{3} - x^2 - 2x + c$ B. $x^4 - x^3 + x^2 + c$ C. $x + \dfrac{x^4}{3} - \dfrac{x^3}{4} - x^2 + c$
5. Integrate $1/x^5$ wrt x. A. $\dfrac{x^6}{6} + c$ B $\dfrac{x^{-4}}{-4} + c$ C. $\dfrac{x^{-4}}{5} + c$

Theory:

1. Find $\int x \sin 2x dx$ 2. Evaluate $\int \dfrac{dx}{x(x + 2)}$

WEEK 3

TOPIC : INTEGRATIO*N [*INDEFINITE INTEGRALDEFINITE INTEGRAL AND AREA UNDER CURVE]

The process of reversing differentiation is called Integration. If $\mathbf{dy/dx = 3x^2}$, then y could be $\mathbf{x^3}$**,** as the derivative of $\mathbf{x^3}$is $\mathbf{3x^2}$.

We say that x^3 is an integral of $3x^2$ with respect to x. The symbol for integration sign is given by $\int$. The expression to be integrated is put between the $\int$ sign and dx.

$\int$ **3x2** dx could be $\mathbf{x^3}$

Since differentiating any constant gives zero, the following also have derivation $\mathbf{3x^2}$.

$\mathbf{X^3 + 2,\ x^3 + 4.5,\ x^3 - 17}$etc

In general, any function of the form x3 + c, where c is the constant has derivative of $\mathbf{3x^2}$

Hence, $\int \mathbf{3x^2\, dx = x^3 + C}$**. C** is called constant of integration. Because we do not know the actual or definite value of C, this is called ***INDEFINITE INTEGRAL.***

Let $y = x^{n+1}/ n+1$, ***Differentiating*** $dy/dx = (n+1)\, x^{n+1}/n+1 = x^n$

Reversing this,

$\int \mathbf{x^n\, dx = x^{n+1}/n+1 + C}$

$\int \mathbf{kxn\, dx = K^{xn+1}/n+1 + C}$

To Integrate a sum, integrate each term as this is similar to differentiating a sum.

Examples:

Evaluate (i) $\int(2x^3 + 3x^2 - 4)\, dx =$

$2/4x^4 + 3/3x^3 - 4x + C$

½ x2 + x3 – 4x + C

Evaluate $\int(4t^3 + 2t^2 + ½\ t^2)$ dt
4/4 t4 + 2/3 t3 + ½ × ½ t^2 + C
t^4 + 2t3 + ¼ t2 + C.

Integration of basic Trigonometric functions
Consider the table below for differentiating trigonometric functions.

Y	sinKx	Cos Kx	Tan Kx
dy/dx	kcoskx	-ksinkx	k/(coskx)^2

Consider a similar table as the one given above

Y	Sin x	Cos x	Tan x		
$\int ydx$	Cosx+C	Sinx +C	Tanx +C		

Y	Sinkx	Coskx	Tankx
∫ydx	-1/k coskx + C	1/k sinkx + C	1/k tankx + C

Example
∫(cos2x + sin3x) dx
∫cos2x dx + ∫ sin3x dx
½ sin2x + -1/3 cos3x
½ sin2x – 1/3 cos3x + K.
Sometimes,the value of the constant C can be found, if extra information is given.
If dy/dx = x2+2x-3, find y in terms of x given that x = 1, y =4
Y =∫(x2 +2x-3)dx
Y =x3/3 + 2x/2 – 3x + C
Y = x3/3 +x2 -3x + C
Putting x=1,and y = 4
4 = 1/3 + 1 – 3 + C. hence; C = 5 2/
;. 1/3x3 – x2 – 3x + 5 2/3.

If dy/dx = 2Cos3x, find y given that y =2 and x = 1/6∏
∫2Cos3x = 2/3Sin3x + C
2 = 2/3Sin3(∏/6) + C = 2/3 Sin∏/2 + C
Multiplying by ½ ∏ (180/∏) = 90^0
Sin ½ ∏ - Sin 90^0 = 1
2 = 2/3 ×1 + C
C = 4/3
Y = 2/3Sin3x + 4/3 .

Evaluation:
Evaluate these indefinite integrals
1. ∫(y2 – 7y)dy
2. ∫(3x2 – 2x – 1)dx
3. ∫(Cos4x)dx
4. ∫(3cos2x + 4sin3x)dx
5. If dy/dx = 4x2 + 1 and y = 2 when x =3, find y in terms of x
6. If dy/dx = 2Sin1/3x and y = 4 when x =∏m, find y

DEFINITE INTEGRALS

In this part, the constant is removed. If a definite integration is performed, the function is evaluated between the values called **limits. Upper and lower ie**$\boldsymbol{\int_{-2}^{3}(x2)dx}$

Example: Evaluate $\int_{2}^{3}(3x2)dx$

$\frac{3x3}{3}$ =x^3 + C , (3^3 + c) – (2^3 + c)

(27 + C) – (8 + C)

27 + C -8 – C

19.

$\int_{2}^{3}(3x2)dx$ = 19.

2. $\int_{0}^{\pi 1/4}(Cos2x)dx$ = { ½ Sin$2x$ π}$\pi/4$

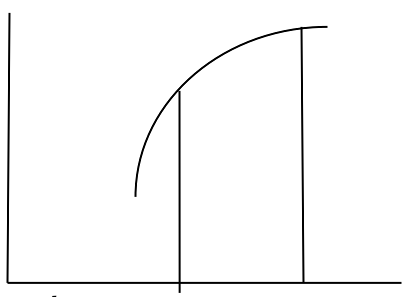
(½ sin$2x1/4$ $\pi/4) - (\frac{1}{2}sin2\times$ 0)

½ sin$\pi\frac{}{2} - \frac{1}{2}sin0. since \frac{1}{2}\pi = 0, andsin901alsosin0 = 0$

$\int_{0}^{\pi/4}(Cos2x)dx$ = ½

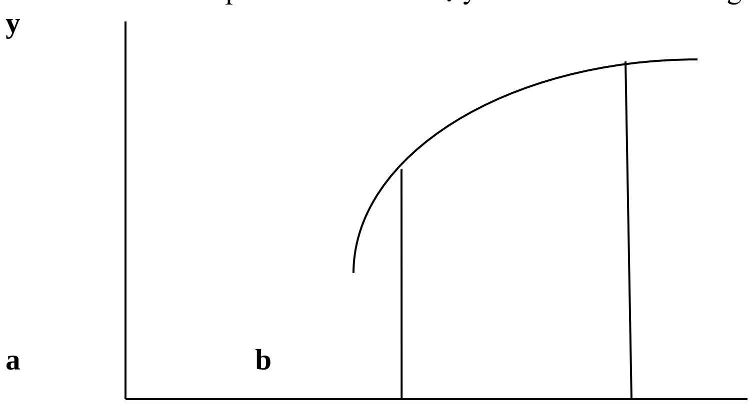
Area under curve using Definite Integral

Given in the diagram, the area between the curve and the x axis from x = a and to x = b. The area is given by

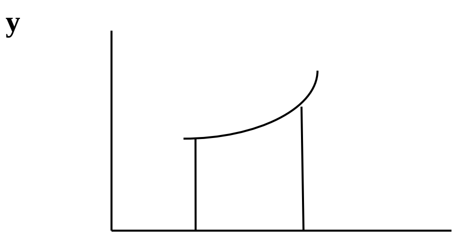


Area = $\boldsymbol{\int_{a}^{b} y\ (dx)}$

The area can be explained as: Area = ∫ y × dx = Sum of height of rectangle × width of rectangle



Ex 1: Find the Area between the curve y = x3 – x and the x axis when x =2 and x = 4.



0 2 4 x

$$\int_2^4 (x3 - x)dx$$

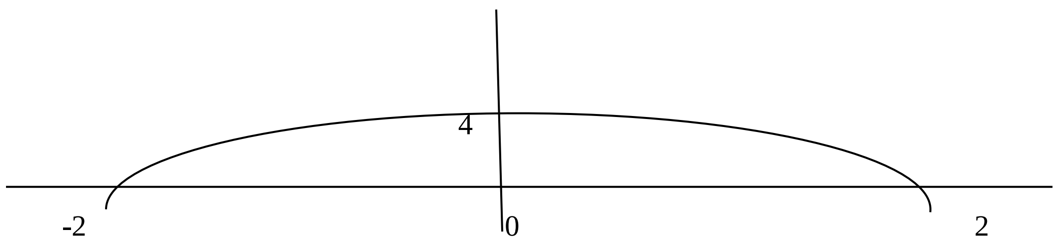
$$(\frac{1}{4}x4 - \frac{1}{2}x2)4$$

{ ¼ $(4)^4$ – ½(4)2} – { ¼ X 2^4 – ½ $(2)^2$}

64 – 8 -4 + 2 = 54 UNITS.

Ex 2 : Find the area in the diagram shown below

$y = 4 – x^2$



∫(4 – x2)dx
$\{4x – x3/3\}_2{}^2$.
$4(2) – 2^3/3$ - 4(-2) – $(-2)^3/3$
(8- 8/3) – (-8 + 8/3)
8 – 8/3 + 8 – 8/3
32/3.square unitrs

Evaluation:
Find the area between the values of **$y = 3x^2$** for x is between 2 and 4
Find the area between the curve $y = x^2 + 3$ At the xs axis and when x = -1 and x = 3.
1. $Find\ the\ area\ enclosed\ between\ the\ curvey = 17 - 2x -x^2$ and the line y = 2.

General Evaluation:
1. ∫(3x – 1)(x + 2) dx
2. ∫5cos4x (dx)
3. $\int_{\bar{\pi}1/8}^{1/6} sin2x\ (dx)$

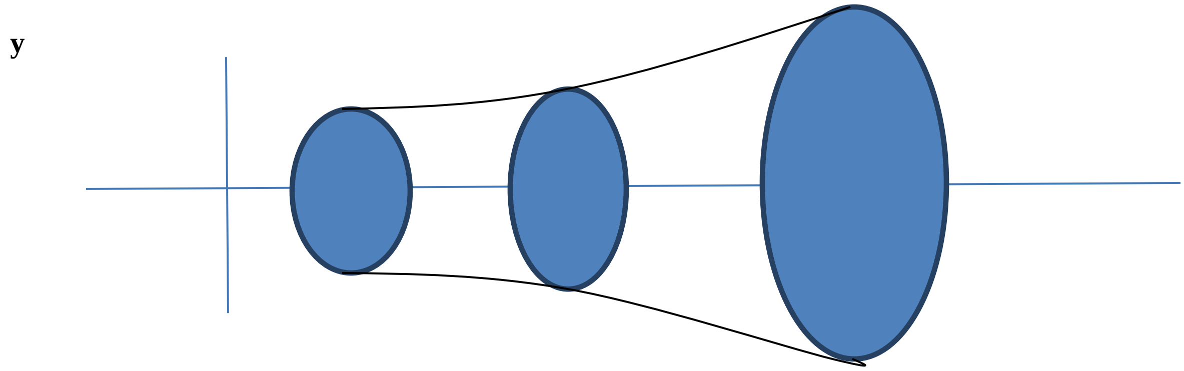
Weekend Assignment:
1. Evaluate $\int_1^3 (x2 - 1)dx$ A. 2/3 B. -2/3 C. -6 2/3 D. 6 2/3
2. Evaluate$\int_2^3 (x2 - 2x)dx$ A. 4 B. 2 C. 4/3 D. 1/3
3. Evaluate$\int_0^{\bar{\pi}1/2} (sin2x)dx$ A. - ½ B. 1 C. -1 D. 0
4. Find the area enclosed by by the curve $y = x^2$, X = 0 and X = 3 A. 9 B. 7 C. 5/2 D. 5
5. Given y = 3x -2, x=3, x=4. Find the area under the curve A. 4/3 B. 17/2 C. 6 D. 3

Theory
1. Find the area enclosed between the curve $y =x^2 + x -2$ and the x axis
2. Find the area enclosed by the curve $y = x^2 – 3x + 3$ and the y = 1.

WEEK 4

TOPIC : APPLICATION OF INTEGRATION II : SOLID REVOLUTION AND TRAPEZOIDAL RULE

A solid whichb has a central axis of symmetry is a ***solid of revolution.***For**example, a cone, a cylinder , a vase etc.**



Consider the area under a portion AB of the curve y = f(x) revolved about the x axis through four right angle or 3600, each point of the curve describes a circle centered on the x axis. A solid revolution can be thought of as created in this way with the circular plane ends cutting the x – axis at x = a and x = b.

Let v be the volume of the solid for x = a up to an arbitrary value of x between a and b. Given abincreament dx in x , and y takes an increamentdy and v increases by dv.

The figure shows a section through the x axis, from this it is seen that the slice dv of bthickness dx is enclosed between two cylinders of outer radius y +dy and inner radius y .

Then ,$\pi y2dx < dv < \pi (y + dy)2dx$;

With appropriate modification, if the curve is falling at this point.

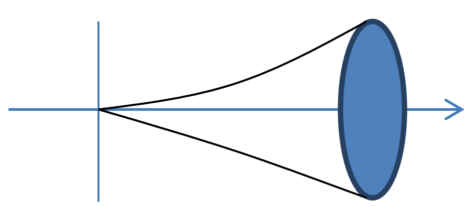
$\pi\, y2 < dv/dx < \pi\, (y + dy)2$

if dx $\rightarrow$ 0, dy $\rightarrow$ 0 as dv/dx $\rightarrow$ dv/dx

: . dv/dx = $\pi y2$ or V = $\int_a^b \pi\, y2dx$

Where y = f(x) and v = volume of solid revolution of the curve where y = f(x) is rotation completely and x – axis between limits x = a and x =b.

<u>Examples;</u>The portion ofthe curve $y = x^2$ between x = 0 and x = 2 is rotated complrtely around the x axis, find the volume of the solid generated?



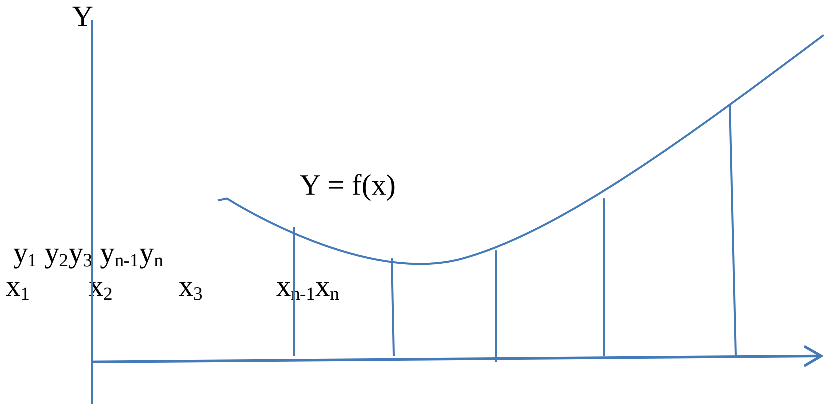
V = $\int_0^2 \pi y2dx$

= $\int_0^2 (\pi\, x2)2\, dx$

V = $\pi \int \pi x4\, dx$

V = π $\int_0^2 x4dx$ = π $\left[\frac{x5}{5}\right]2$
Put x = 2 and x = 0 then substitute into the expression above
V = π $\left(\frac{32}{5}\right) = \frac{32\pi}{5} units of volume.$

THE TRAPEZOIDAL RULE
There are many definite integrals which can't be evaluated and thus required advance techniques e.g
$\int_0^1 \frac{dx}{x}, \int_0^1 \frac{dx}{1+x2}$,etc
We can find an approximate value for such integralsbyb finding the area approximately. There are many methods methods of doing this and such methods include the ***<u>Trapezium rule.</u>***



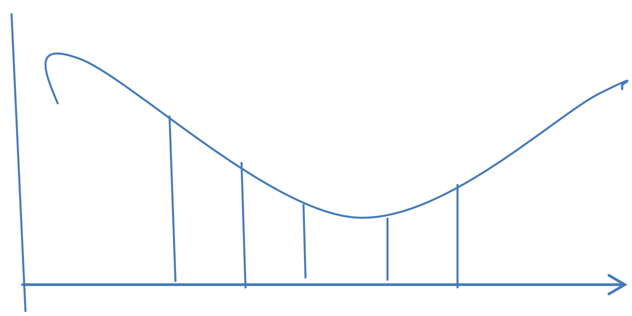
$\int_{x1}^{xn} f(x)dx$ = ½ (y_1 +y_2)h + ½ (y_2 +y_3)h + ½ (y_{n-1} +y_n)h
½ h(y1+2y2+2y3+……….+2yn-1 + yn)
½ (width of each trap.) × (first ordinate + last ordinate)+ 2(sum of all other ord.)

F(x)dx = ½ h{y1 + yn} +2{y2 +y3 + …Yn}.

Example
Find the approximate value of $\int_1^3 \frac{dx}{x}$ at interval 0.5

X	1	1.5	2	2.5	3.0
Y = 1/x	1	0.67	0.5	0.4	0.33

Applying the rule;
{ ½ . ½ { (1 +0.33) + 2 (0.67 + 0.5 + 0.4)}
¼ {(1.33) +2(1.57)}
¼ (4.47) = 1.12.



1 0.67 0.5 0.4 0.33

Ex (2). Make a table of value of y for which y = $\frac{2}{\sqrt{x2-1}}$ for which x =2 t0 x = 3 at interval of 0.2.

$$\int_2^3 \frac{dx}{\sqrt{x2-1}}$$

X	2	2.2	2.4	2.6	2.8	3
X2-1	3	3.84	4.76	5.76	6.84	8
$\sqrt{x2}-1$	1.732	1.956	2.182	2.4	2.615	2.828
$\frac{1}{\sqrt{x2}-1}$	0.5754	0.5703	0.4583	0.4167	0.3824	0.3536

Using the rule.
$\int_2^3 \frac{dx}{\sqrt{x2-1}}dx$= ½ *0.2 (0.5774 +0.3536 + 3.5354)
0.44664 *2
0.89 correct to 2 dp

APPLICATION OF INTEGRATION TO KINEMATICS

If the ve;locityis given as a function of time, the displacement is the integral of the velocity function with respect to the time .
ds/dt = f(t)
then S = $\int f(t)dt$
= f(t) + C
Similarly, if the acceleration is a function of time, the velocity is the integral of the acceleration function.
Ex. A particle is projected in a straight line from O until a speed of 6m/s is attained. At time t secs.Later,its acceleration is (1 + 2t) m/s^2 for the value of t = 4. Calculate for the particle (i) its velocity (ii) its distance from O

dv/dt = 1+ 2t
v = $\int(1+2t)dt$ = t + t^2 + C
when t = 0
v = 6m/sand c = 6.
V = (t^2 + t + 6) m/s
When t = 4, v = 16 + 4 + 6 = 26m/s.
(ii) distance (s) = ds/dt = t2 + t + 6
S = $\int(t2+t+6)dt$
S = $\{t3/3 + 16/2\ 6t\}^4$
$\frac{64}{3}+\frac{16}{2}+24$ = 160/3
$\frac{160}{3}$m.

Evaluation

1. Find the area enclosed byb y = x2 – x -2 and the x axis
2. find the area under the curve y = x2/3 between x = 2 and x = k is 8 times the area under the same curve between x = 1 and x =2, hence find the value of k.

GENERAL EVALUATION

1. A particle moves in astraight line from O until the initial velocity was 2m/s. its acceleration is given by (2t -3)m/s2. Calc. (i) its velocity after 3 secs. (ii) the distance from O when it is momentarily at rest.
2. Find the volume of solid revolution when a is the region bounded by the cuerve y = 2x. and the ordinate at x = 2,and x = 4 and the x axis is revolved by 2π.

Reading Assignment :F/Matrhs Project, pg 47 – 63

WEEKEND ASSIGNMENT

1. Integrate 2√x A. $\frac{4x3/2}{3}+\ C$ B.4x$^{3/2}$ + C C. $\frac{4x}{3}$ + C D. $\frac{\frac{3}{2}x4}{4}$

2. Integrate $\frac{1}{3\sqrt{x}}$ A. $\frac{3x2/3}{2}$ + C B. x$^{3/2}$ + C C. x^{2} + C D. $\frac{2x3/2}{3}$ + C

3. The gradient of a curve is 6x + 2 and it passes through the point (1,3), find its equation A. 3x^{2} - 2x + 2 B. 3x^{2} -2x -2 + 2x + 2 C. 3x^{2} – 2x + 2 D. 3x^{2} – 2x -2

4. Evaluate $\int(x-1)(x2+2)$ A. $\frac{x4}{4}-\frac{x3}{3}$ + x2 – 2x + C B. x3/3 – x3/2 + x2 + 2x +C C. x3/3 + x2/2 +x3-2x + C D. x4/4+x3/3-x2+2x+C

5. Eval. $\int_{-1}^{2} 3x2$ A. 9 +C B. 8/3 + C C. 24 D. 18 + C

THEORY

1. Evaluate $\int_{3}^{6}\frac{2x6-5}{x2+2}dx$

2. Using trapezoidal rule, with ordinate x = -3,-2,-1,0,1,2,3 and 4. Calc correct to 3 dp an approximate value of $\int_{-3}^{4}\frac{6x}{x2+2}dx$

WEEK 5

TOPIC: REGRESSION LINE AND CORRELATION COEFFICIENT

SCATTER DIAGRAM

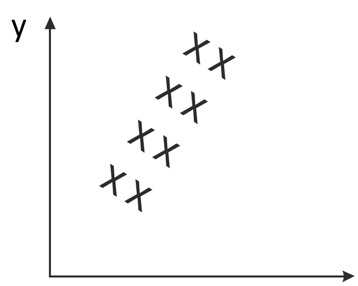
Definition: a scatter diagram is a graphic display of bivariate data. A bivariate data involves two variables

TYPES OF SCATTER DIAGRAM:

Linear positive correlation.

A positive correlation between two variables x any y means that in general, increase in x is accompanied by increase in y. The regression line has a positive slope.

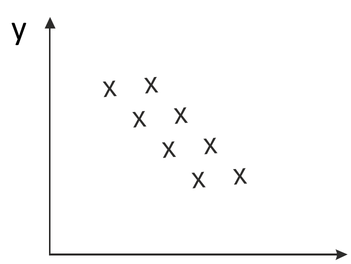
X



Linear negative correlation

A negative correlation between x and y means that an increase in x is accompanied by a decrease in y, negative correlation has a negative slope.

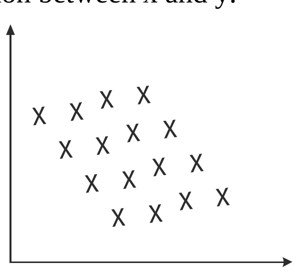
x



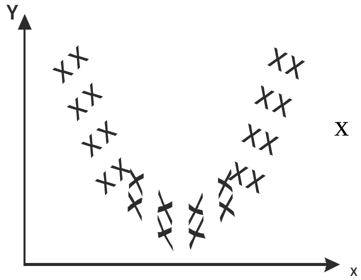
Zero Correlation:

There is no apparent association between x and y.

y



Non Linear Correlation:
Most of the points lie on or near a curve which is parabolic in shape. The parabolic curve is called a regression curve.



REGRESSION LINE OR LINE OF BEST FIT OR THE LEAST SQUARES LINE
There are two variables where one is dependent and the other is independent variable. The regression line can be fit using scatter diagram method and the least squares method.

LEAST SQUARES METHOD: If x is independent variable and y dependent variable, that is y on x. then :The equation of the regression line is written as $y = ax + b$
Where a is the slope and b is the y – intercept. Given two sets of variables x and y it can be deduced that

$$\mathbf{a = n \sum xy - \frac{\sum x \sum y}{\sum x^2 - (\sum x)^2}}$$

$b = \bar{y}\, a - a\bar{x}$

Where $\bar{x} = \frac{\sum x}{n}$

$\bar{y} = \frac{\sum y}{n}$

Example: use the least square method to fit a regression line of y on x for the following data

X	3	5	6	9	11	14	15	18
Y	2	3	5	7	10	12	13	17

Find value of y when x = 8

SOLUTION:

X	y	Xy	x^2
3	2	6	9
5	3	15	25
6	5	30	36
9	7	63	81
11	10	110	121
14	12	168	196
15	13	195	225
18	17	306	324
$\sum x = 81$	$\sum y = 69$	$\sum xy = 893$	$\sum x^2 = 1017$

$$a = \frac{n\sum xy - \sum x \sum y}{n\sum(x^2) - (\sum x)^2} = \frac{8\,(893) - 81 x\, 69}{8\,(1017)}$$

$$a = \frac{7144 - 5589}{8136 - 6561} = \frac{1555}{1575}$$

$$a = 0.9873$$

$$\bar{x} = \frac{\sum x}{n} = \frac{81}{8} = 10.125$$

$\bar{y} = \frac{\sum y}{n} = \frac{69}{8} = 8.625$

$b = \bar{y} - a\bar{x}$

$b = 8.625 -- 0.9873 (10.125)$

$= 8.625 – 9.996$

$b = -1.37$

$y = ax + b$

$y = 0.9873x – 1.37$ (regression line of y on x)

When $x = 8$

$y = 0. 9873 (8) – 1.37$

$y = 6.5284 \sim 6. 5$

EVALUATION

Use the least square method to fit a regression line of y on x for the following data

X	1	4	5	7	8	10	12	16	19	20
Y	2	3	4	5	7	8	10	15	20	18

Use the line obtained to find the value of y when x = 9

CORRELATION COEFFICIENT

DEFINITION:

The correlation coefficient determines the amount or degree of linear relationship between two variables. The correlation coefficient is represented by r

The characteristics of r are as follows:

1. The value of r is the same irrespective of the variable labelled x or y.
2. the value of r satisfies the inequality $-1 \leq x \leq + 1$
3. if r is close to +1, the variables are highly positively correlated. If r is close to -1 then, x and y are highly negatively correlated. If r is close to zero, the correlation between x and y is very low. There is no correlation between x and y when r = 0

There are two methods of obtaining the correlation coefficient.

1. Pearson's coefficient of correlation or product moment correlation coefficient
2. Rank correlation coefficient.

RANK CORRELATION COEFFICIENT: It is also known as Spearman's rank correlation coefficient and defined as :

$$\mathbf{r_k = 1 - \frac{6 \sum D^2}{n(n^2 -1)}}$$

As the name implies, the variables (if not ranked) can be ranked in ascending order or descending order. Where there are ties, the average is used as the rank.

Where D is the difference between the pairs of variables and n is the number of variables. D = Rx – Ry

Example:

The table below gives the examination marks of 10 students in mathematics and history.

Maths	51	25	33	55	65	38	35	53	61	44
History	20	65	25	36	51	50	77	31	60	5

A Calculate the rank correlation coefficient

b) Comment briefly on your result

SOLUTION:

MATHS (x)	HISTORY (y)	Rx	Ry	D	D^2
51	20	5	9	-4	16

25	65	10	2	8	64
33	25	9	8	1	1
55	36	3	6	-3	9
65	51	1	4	-3	9
38	50	7	5	2	4
35	77	8	1	7	49
53	31	4	7	-3	9
61	60	2	3	-1	1
44	5	6	10	-4	16

$\sum D^2 = 178$

$$r_k = 1 - \frac{6\sum D^2}{n(n^2 - 1)}$$

$$= 1 - \frac{6 \times 178}{10(10^2 - 1)}$$

$1 - 1068/990$

$= 1 - 1.178 = -0.078$

There is a very low negative correlation between the marks obtained in mathematics and history.

EVALUATION:

The table below shows the marks obtained by ten students in both theory (x) and practical (y) examination.

X	50	70	85	35	60	65	75	40	45	80
Y	45	55	75	40	50	60	70	35	30	65

Calculate the rank correlation coefficient between x and y comment on your result.

PEARSON'S CORRELATION COEFFICIENT: It is fully called Pearson's product moment correlation coefficient. It is simple to calculate and it does not recognise any of the variables as independent or dependent. It is obtained using the formula below.

$$\mathbf{r = \frac{n\sum xy - \sum x \sum y}{\sqrt{[n\sum(x^2) - (\sum x)^2][n\sum(y^2) - (\sum y)^2}}}$$

Example:

Calculate the product moment correlation coefficient for the following data

X	2	4	7	9	11
Y	1	2	3	7	9

Comment on your result.

SOLUTION:

X	Y	XY	X^2	Y^2
2	1	2	4	1
4	2	8	16	4
7	3	21	49	9
9	7	63	81	49
11	9	99	121	81
$\sum x =$ 33	$\sum y =$ 22	$\sum xy =$ 193	$\sum x^2 =$ 271	$\sum y^2 =$ 144

$r = \dfrac{5 \times 193 - 33 \times 22}{\quad}$

$\sqrt{[5(271) - (33)^2][5(144) - (22)^2]}$

$r = \dfrac{965 - 726}{\sqrt{266 \times 236}}$

$r = \dfrac{239}{250.55}$

r = 0.9539. r = 0.95 (approximately to 2 s.f)

Comment: The relationship between x and y is highly positive.

EVALUATION: The following data are the marks obtained by five students in statistics (X) and mathematics(Y). Calculate the product moment correlation coefficient and comment on your result.

X	33	36	42	52	40
Y	42	46	38	62	52

GENERAL EVALUATION/REVISIONAL QUESTIONS

1. If Cos A = 24/25 and Sin B= 3/5, where A is acute and B is obtuse, find without using tables, the values of (a) Sin 2A (b) Cos 2B (c) Sin (A-B)
2. Use the addition formula to find the values of the following
(a)Sin 75^0 (b) cos 75^0 (c) tan 45^0
3. Calculate the Product moment correlation coefficient and the Spearman's rank correlation coefficient.

X	50	45	43	30	30	43	23	43	25
Y	12	13.5	14	11	12	15	13.5	12	14

READING ASSIGNMENT: Read correlation and regression.Page313–320. Further Mathematics project 2.

WEEKEND ASSIGNMENT

Use the table below to answer questions 1 and 2.

Height	160	161	162	163	164	165
No of students	4	6	3	7	8	2

1 The mean of the distribution is
(a) 4875.1 cm (b) 4001.2 (c) 3571.0cm (d) 162.2 cm (e) 129.2cm

2. The median of the distribution is
(a) 160 (b) 162 (c) 163 (d) 164 (e) 165

3. Calculate the standard deviation of 3,4, 5,6,7,8,9
(a) 2 (b) 2.4 (c) 3.6 (d) 4.0 (e) 4.2

4. Calculate the mean deviation of 6 , 8 , 4 , 0 , 4
(a) 4 .0 (b) 3.6 (c) 3.0 (d) 2. 8 (e) 2 . 1

5. The table below shows the rank Rx and Ry of marks scored by 10 candidates in an oral and written tests respectively. Calculate the spearman's rank correlation coefficient of the data.

R_x	1	2	3	4	5	6	7	8	9	10
R_y	2	3	4	1	6	5	8	7	10	9

(a)51/55 b) 6/55 c)49/55 d)54/55 e) 61/55

THEORY

1 The distribution of marks scored in statistics and mathematics by ten students is given in the table below:

Maths(x	11	20	23	42	48	50	57	64	80	90
Stat(y)	26	23	35	46	44	50	50	58	68	70

a) Plot a scatter diagram for the distribution
b) Draw an eye- fitted line of best fit
c) Use your line to estimate the students marks in statistics if his mark in maths is 40

2. The table below gives the marks obtained by members of a class in maths and physics examination

STUDENTS	A	B	C	D	E	F	G	H	I	J
Maths	85	75	59	43	74	69	62	80	54	63
Physic	92	72	62	48	85	73	46	74	58	50

a) Calculate the product moment correlation coefficient.
b) Comment on your result.

WEEK 6 PROBABILITY DISTRIBUTION

- BINOMIAL PROBABILITY DISTRIBUTION
- POISSON PROBABILITY DISTRIBUTON

Probability distribution deals with theoretical probability model based on the randomness of certain natural occurrences. The binomial and Poisson distribution are discrete distribution

BINOMIAL DISTRIBUTION

This arises from a repeated random experiment which has two possible outcomes.

The two possible outcomes of the random experiment are usually called success and failure.

Prob(success) = P, Prob(failure) = q

Since the two events are complementary, hence p+ q = 1 or p = 1-q, q = 1 – p

The probability of success or failure of an event is the same for each trials and does not influence the probability of success or failure of another trial of the same event.

:. Binomial distribution of n trails and r required outcome(s) is defined as :

$$pr(x = r) = {}^{n}C_{r}P^{r}q^{n-r}$$

$$\text{when } {}^{n}C_{r} = \frac{n!}{(n-r)!\, r!.}$$

The binomial distribution is suitable when the number of trials is not too large.

Example:

1. Find the probability that when two fair coins are tossed 5 times a head and a tail appear three times.

Solution:

Two fair coins = (HT, TH, TT, HH) = 4

Prob (a head and a tail) = $^2/_4 = ½$

i.e $p = ½$, $q = ½$ $(p + q = 1)$

$n = 5, r = 3$.

:. $P(x = r) = {}^nCrp^rq^{n-r}$

$p (x = 3) = {}^5C_3 (½)^3 (½)^{5-3}$

$p (x = 3) = 10 \times {}^1/_8 \times ¼ = {}^{10}/_{32} = {}^5/_{16}$

$p (x = 3) = 0.3125$.

2. It is known that 2 out of every 5 cigarettes smokers is a village have cancer of the lungs. Find the probability that out of a random sample of 8 smokers from the village, 5 will have cancer of the lungs.

Solution

Prob(a smoker has cancer) = $^2/_5$i.e $p = {}^2/_5$

Prob(a smoker doesn't have cancer) = $1 – {}^2/_5 = {}^3/_5$

:. $q = {}^3/5$

$n = 8, \quad r = 5$

Prob(x = -5) $= {}^8C_5 ({}^2/_5)^5 ({}^3/_5)^3$

$$= 56 \times \frac{32}{3125} \times \frac{27}{125} = \frac{48384}{390625}$$

Prob (x = 5) = 0.124.

EVALUATION

Find the probability that when a fair six-faced die is tossed six times, a prime number appears exactly four times.

POISSON DISTRIBUTION: The Poisson distribution is more suitable when the number of trials is very large and probability of successes is small. It is defined as:

$$\Pr(x) = \frac{\lambda^x e^{-\lambda}}{x!} \quad , x = 0, 1, 2, 3,$$

Where $\lambda = np$ $\quad\quad\quad\quad$ $e = 2.718$

P = probability of success, n = number of trials.

Example:

If 8% of articles in a large consignment are defective, what is the chance that 30 articles selected at random will contain fewer than 3 defective articles?

Solution

P = 8/100 = 0.08, n = 30

$\therefore \quad \lambda = np = 0.08 \times 30 = 2.4.$

Prob(fewer than 3) i.eprob((0) + prob (1) + prob (2)

$$\text{Prob } (x = 0) = \frac{2.4^{o} \times e^{-2.4}}{0!} = 1 \times {}^{e-2.4}$$

$$\text{Prob}(x = 1) = \frac{2.4^{o} \times e^{-2.4}}{1!} = 2.4 \times e^{-2.4}$$

$$\text{Prob}(x = 2) = \frac{2.4^{o} \times e^{-2.4}}{2!} = 2.88 \times e^{-2.4}$$

$$\text{Prob}(x < 3) = e^{-2.4} + 2.4 \times e^{-2.4} + 2.88 \times e^{-2.4}$$

$$= e^{-2.4} (1 + 2.4 + 2.88)$$

$$= e^{-2.4} \times 6.28.$$

EVALUATION

The probability that a person gets a reaction from a new drug on the market is 0.001. If 200 people are treated with this drug. Find approximately, the probability that:

(i) exactly three persons will get a reaction
(ii) more than two person will get a reaction

Properties of Binomial and Poisson Distribution.

Binomial

It assigns probability to non-occurrence of events i.eProb($x = 0$)

Mean $\mu = np$

Standard deviation, $r = \sqrt{npq}$

Variance $ð^2 = npq$

Poisson

It assigns probability to non-occurrence of events i.eProb($x = 0$)

Mean $\mu = \lambda = np$

Standard deviation, $ð = \sqrt{\lambda} = \sqrt{np}$

Variance, $ð^2 = \lambda = np$

Example:

1. In the probability of tossing a fair coin three times, a head shows up twice. Find the mean and standard deviation.

Solution

$n = 3$ Prob(a head) $= ½$, i.e $p = ½$, $q = ½$

I. Mean $\mu = np = 3 \times ½ = 3/2$

II. Standard deviation :$r = \sqrt{npq} = 3 \times ½ \times ½ = ¾$

Example

2. 0.2% of the cooks produced by a machine were found to be defective. If there are 1000 corks, find the mean and standard deviation?

Solution

$P = 0.2\% = 0.002$.

$N = 1000$

I. mean $\mu = \lambda = np = 1000 \times 0.002 = 2$

II. $r = \sqrt{np} = \sqrt{2}$

EVALUATION

In an examination, 60% of the candidates pass. If 10 candidates were sampled. Find the mean, standard deviation and variance of the candidates.

GENERAL EVALUATION

1. The probability that a person gets a reaction from a new drug in the market is 0.001. If 2000 people were treated with this drug, find the mean and standard deviation.
2. 1. In an examination, 60% of the candidates passed. Use the binomial distribution to calculate the probabilities that a random sample of 10 candidates contain exactly 2 failures.

READING ASIGNMENT

Read probability distribution, further math. Project 3 from page 198-201.

WEEKEND ASSIGNMENT

1. What is the variance of a binomial distribution?

   (a) np (b) $\sqrt{npq}$ (c) npq (d) p^2

2. The mean (μ) of a poisson distribution is the same as

   (a) Standard deviation (b) variance (c) mean (d) mean deviation

3. If number of trials is 100 and probability of success is 0.0001, what is the variance of this distribution?

   (a) 0.00999 (b) 0.1 (c) 0.01 (d) 0.001

4. If the birth of a male child and that of a female child are equiprobable. Find the probability that in a family of five children exactly 3 will be male. (a) 16/5 (b) 5/16 (c) 5/32 (d) 5/21

5. If an unbiased die is thrown repeatedly, what are the chances that the first, six to be thrown will be the third throw? (a) 25/216 (b) 1/6 (c) 25/36 (d)25/31

THEORY

1. 20% of the total production of transistors produced by a machine are below standard. If a random sample of 6 transistors produced by the machine is taken, what is the probability of getting

   (i) exactly 2 (ii) exactly 1 (iii) at least 2 (iv) at most 2 standard transistors?

2. A fair die is thrown five times. Calculate correct to 3 decimal places, the probability of obtaining

(a) at most two sixes (b) exactly three sixes

WEEK 7 STATICS

- Definition of Concepts
- Resultant of two forces
- Components resolution of forces

DEFINITION OF CONCEPT

Statics is the study of bodies which remain at rest under the action of given forces.

Mass: This is the quantity of matter contain in a body. Mass of a particular body does not change and the standard unit is kilogram.

Force: Force is that action which tends to change the state of rest or uniform motion of a body in a straight line. It's a vector quantity sine it has magnitude and direction. The unit of force is Newton.

:. F = Ma

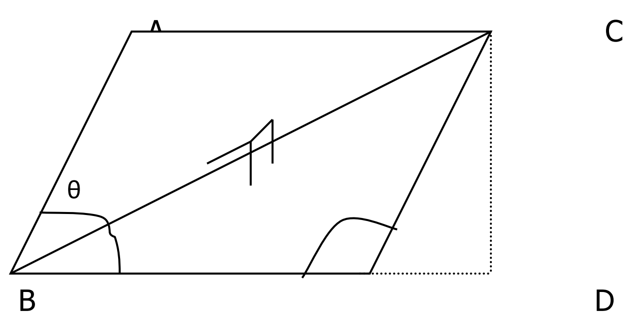
Where M = Mass, a = acceleration.

Composition of Forces: Two or more concurrent forces can be combined to obtain a single force. Therefore, resultant force is the force produced or obtained when two or more concurrent forces are combined.

A force can be resolve by

I Graphical Method II. Analytical Method.

Analytical Method: The parallelogram law of composition of two forces is used to find the resultant force of two or more forces. Hence, parallelogram law states that if two forces acting at a point are represented in magnitude and direction by two adjacent sides of a parallelogram, then the resultants of the two forces, is represented in magnitude and direction by the diagonal of the parallelogram, drawn from the point of action of the two forces.



R is the resultant force and can be obtained using cosine rule:

$R^2 = P^2 + Q^2 – 2PQ\cos(180 –\theta)$

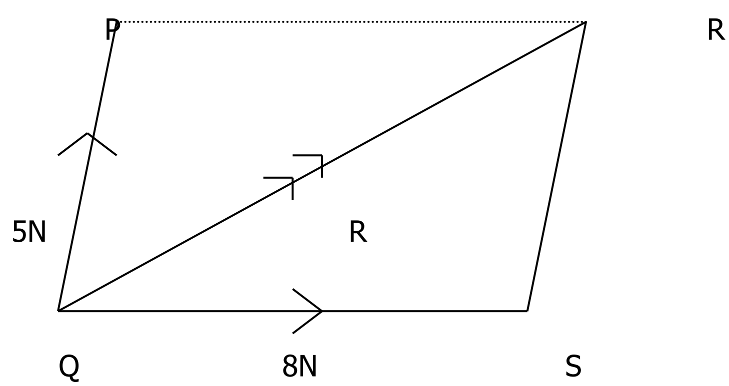
The angle of inclination is ἁ and can be obtained using sine rule or tan.

:. $\text{Tan } ἁ = \frac{CD}{OD}$

$\text{Tan } ἁ = \frac{P \sin \theta}{Q + P \cos \theta}$

Example 1: The angle between two forces of magnitude 8N and 5N is 120^0. Find in N, the magnitude of their resultant.

Solution



Let R be the resultant vectorial force.

$R^2 = P^2 + Q^2 – 2PQ \cos R.$

$= 5^2 + 8^2 – 2 (5 \times 8) \cos 60^o$

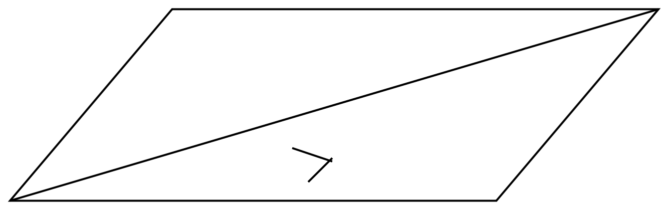
$= 25 + 64 – 80 \times ½$

$= 89 – 40$

$R^2 = 49$

:. $R = \sqrt{49}$ $\quad$ $R = 7N$

2. Calculate, correct to one decimal place, the angle between two forces 20N and 30N if their resultant is 40N.



$R^2 = P^2 + Q^2 - 2PQ \text{ Cos } (180 - \theta)$

$40^2 = 20^2 + 30^2 - 2 (20 \times 30) \text{ Cos } (180 - \theta)$

$1600 = 400 + 900 - 1200 \cos(180 - \theta)$

$1600\text{-}1300 = 1200\text{Cos } (180 - \theta)$

$300 = -1200 \text{ Cos } (180 - \theta)$

$\frac{300}{-1200} = \text{Cos } (180 - \theta)$

$-1/4 = \text{Cos } (180 - R)$

$- 180 - \theta = \text{Cos}^{-1}(^{-1}/_4)$

$180 - \theta = 104.5$

$180 - 104.5 = \theta$

$\theta = 75.5$

:. The angle between them is 75.5^o

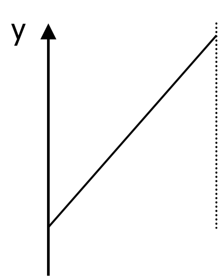
EVALUATION

The angle between two forces of magnitude 8N and 11N is 35^o. Find the magnitude and inclination to the 11N force of resultant force.

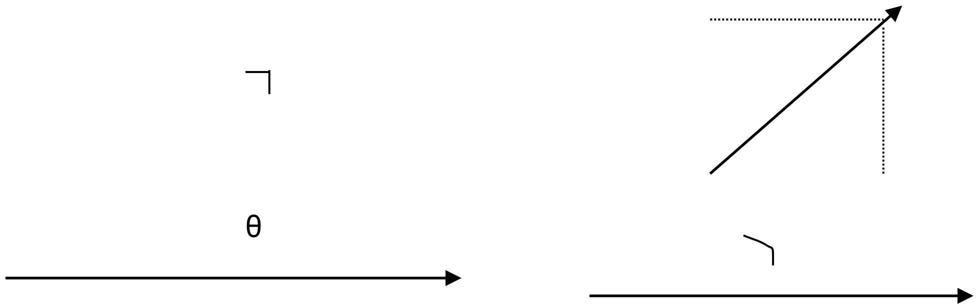
RESOLUTION OF FORCES

A given force can be resolved into two parts and each part is called the resolute. A given force can be resolve in two directions which are perpendicular to each other.

The component along the y axis is called the vertical component and the component along the x- axis is the horizontal component.



pyPy



PxPx

Let Px the horizontal component

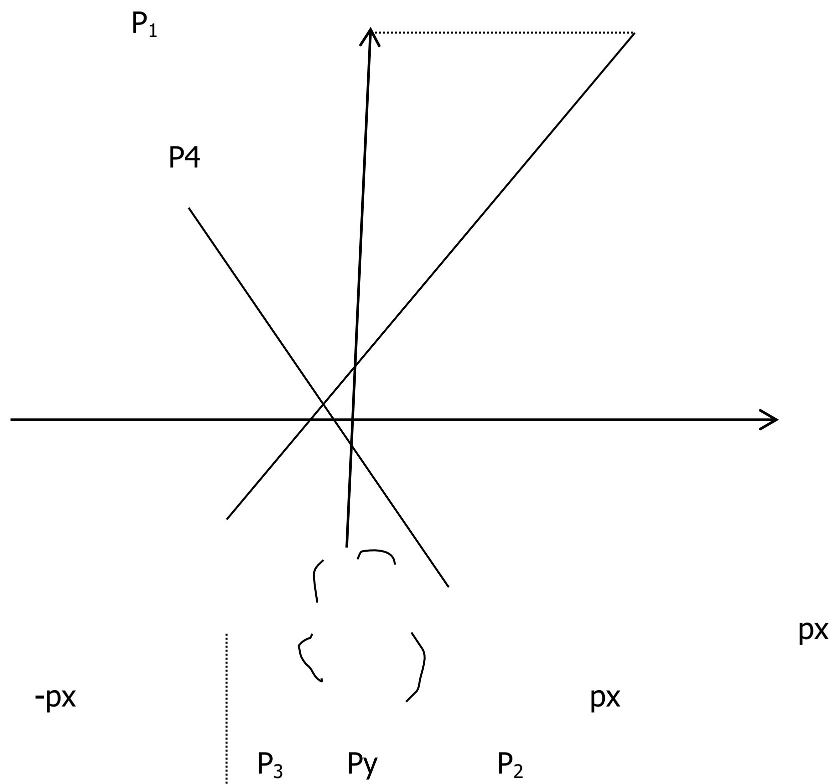
Let py the vertical component.

If p is inclined to the upward vertical

Px Sin θ = $\frac{px}{P}$ Cos θ = $\frac{py}{P}$

P_x = P sin θ. Py = P cos Θ.

Py



py

Force Horizontal Component Vertical Component

P1	P1 Sin θ	P1 Cos θ
P2	P2cos θ	-P2 Sinθ
P3	-P3 cos θ	- P3 Sin θ
P4	- P4 cos θ	P4 sin θ

Resultant of several concurrent forces :the resultant is obtained as :

$$R = \sqrt{(\Sigma P_x)^2 + (\Sigma P_y)^2}$$

Where ΣPx = Sum of horizontal components ΣPy = Sum of vertical components

$$\text{Tan } \theta = \frac{\sum Py}{\sum Px}$$

$$\Theta = \tan^{-1}\left(\frac{\sum Py}{\sum Px}\right)$$

Angle of inclination to the resultant.

Example:

1.The horizontal component of a force p which makes an angle of 50o with the horizontal is 30N. Find the force P.

Solution

$30 = P \times 0.6 + 28$

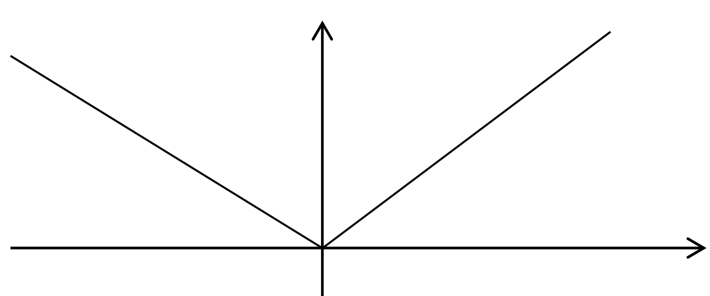
$P = 30/0.6428$

$P = 46.67N$

2.Three forces (2N, 060^0), (4.5N, 180^0) and (5N, 300^0) act on a body of mass 2kg which is initially at rest. Find I the resultant force on the body II the acceleration with which the body begins to move.

Solution

Let P1 = (2N.060^0). P2 = (4.5N, 180^0) P3 = (5N, 300^0)



5N 2N

4.5N

Horizontal Component

$P1 = 2 \sin 60^0 = 1.7321$

$P2 = 4.5 \cos 90^0 = O$

$P3 = 5\cos 30^0 = -4.33$

$\sum px = 1.732 + 0 - 4.33$

$= - 2.598$

$(\sum Px)^2 = 6.7496$

Vertical Components

$P1 = 2 \cos 60^0 = 1$

$P2 = -4.5 \sin 90^0 = -4.5$

$P3 = 5 \sin 30^0 = 2.5$

$\sum py = 1 - 4.5 + 2.5$

$\sum py = -1$

$(\sum py)^2 = 1$

$:. R = \sqrt{(\sum Px)^2 + (\sum Py)^2} = \sqrt{6.7496 + 1} = \sqrt{7.7496}$

$R = 2.78N$

I. Acceleration of the body ; $F = ma$
$M = 2kg$ F = Resultant force = 2.78

$F = ma$

$= 2 \times a$
$a = {}^{2.78}/_{2}$ $a = 1.39ms^{-2}$

EVALUATION

The vertical component of a force F which makes an angle of 35^0 with the horizontal is 45N. Find the force F.

GENERAL EVALUATION

1. The forces of magnitude 35N and 45N act on a particle in the directions 180^0 and 315^0 respectively. Find the resultant of these forces giving:
   a. the magnitude correct to the nearest whole number b. the direction correct to the nearest degree.
1. A vertical force of 6N and a horizontal force of 8N act on a body. Find the magnitude and the inclination of the resultant force to the horizontal.

READING ASIGNMENT

Read Composition and Resolution of Coplanar forces on pages 154 to 165 of further mathematics project III.

WEEKEND ASSIGNMENT

1. Two forces each of magnitude PN are inclined to each other at an angle of 120^0. Find the magnitude of their resultant.(a) P√3 N (b) P2N (c) PN

2. Find the angle between the two forces 5N and 6N if their resultant is 8N.(a) 60^0 (b) 120^0 (c) 180^0

3. A force P of magnitude 60N makes an angle of 40^0 with the horizontal. Use the information to answer questions 3 and 4

3. Find the horizontal component of P (a) 20N (b) 45.96N (c) 38.57N

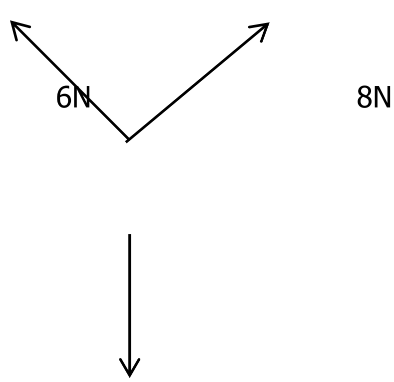
4. Find the vertical component of P (a) 45.96N (b) 38.57 (d) 20N.

5. Find the resultant of forces 8N and 10N inclined at an angle 120^0 to each other.

   (a) 2√61N (b) 61√2N C 39N

THEORY

1. Forces F1 = (10N, 090^0), F2 = (20N, 210^0) and F3 = (4N, 330^0) act on a body at rest on a smooth table. Find, correct to one decimal place the magnitude of the resultant force.
2. Find the magnitude and direction of the resultant of the forces shown in the diagram below:



10N

WEEK 8

STATICS – CONTINUATION

Definition of equilibrium

Condition of equilibrium of rigid body

Application of the condition to solve problems

Lami's theorem and application

Definitions

Equilibrium is when a body remains at rest under the action of given forces.

Translational Equilibrium: The state of equilibrium of bodies which remain at rest under the action of forces have tendency to cause translation.

Condition of Equilibrium

When a block is placed on a table as shown below and force F1 and F2 are applied to the block.

The block remains in translational equilibrium if the magnitude of F1 and F2 are equal.

Since F_1 and F_2 are acting in opposite direction, but have equal magnitudes.

Then, $F_1 = -F_2$

$$F_1 + F_2 = 0$$

Also, the upward force N balances the downward force mg on the block.

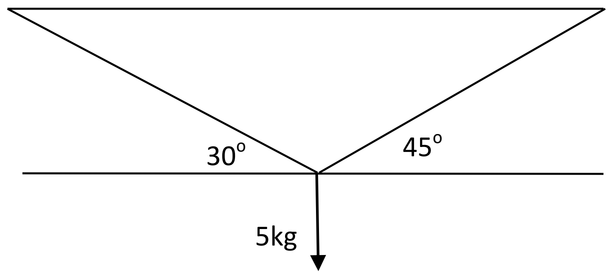
$\therefore \quad N = -mg, N + mg = 0$

Hence, the sum of the vertical components and horizontal components of forces acting on a body in translational equilibrium is equal to zero.

Example

1. A particle of mass 5kg is supported by two light inelastic strings inclined at angles 30^0 and 45^0 respectively to the horizontal. If the system is in equilibrium, calculate the tension in each string.

Solution:



Let the two inelastic strings to be DP and OQ

Resolve each string vertically and horizontally:

Horizontal : Efx $= T_1 \text{Cos } 30^0 + T_2 \cos 45^0 + 0$

Vertical : Efy $= T_1 \text{ Sin } 30^0 + T_2 \sin 45^0 - 50$

:. Efx $= 0.7061T_2 - 0.866T_1 = 0$ equation I

Efy $= 0.7071T_2 + 0.5T_1 = 50$ equation II

Solving equation I and II simultaneously:

$- \quad 1.366T1 = -50$

$T_1 = -50/-1.366 \qquad T_1 = 36.6N$

Substitute T1 in equation I or II using equation 1, $0.7071T_1 - 0.866T_1 = 0$.

$0.7071T_2 = 0.866 \times 36.6$

$$T_2 = \frac{31.6956}{0.7071}$$

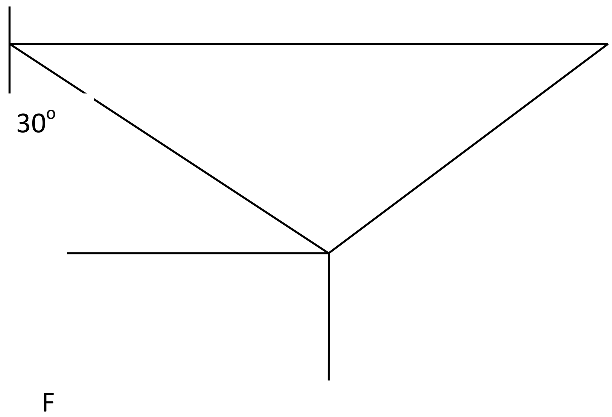
$T_2 = 44.82N$

:. The tensions in the strings are $T_1 = 36.6N$, $T_2 = 44.82N$

2. A particle of mass 5kg is suspended by a light in extensive string which makes an angle of 30^0 with the downward vertical and horizontal forced F. If the system is in equilibrium, calculate

(i) The tension in the string (ii) The magnitude of the force F (take $g = 10ms^{-2}$)

Solution



I using $\sin \theta = \frac{AB}{OA}$

$\sin 60^0 = \frac{5 \times 10}{T}$

$T = 50/\sin 60^0$, $T = 57.74N$.   The tension in the string is 57.74 N

II. Magnitude of F_1 $\tan 30^0 = \frac{OB}{AB}$

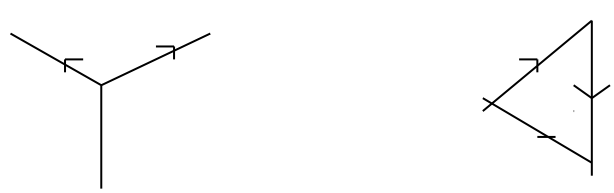
$50 \times \tan 30^0 = F$:. $F = 28.87N$

EVALUATION

1.A street lamp of mass 10kg is suspended at a position) by two wires OP and OQ across a rood such that each wire is inclined at an angle of 80^0 to the upward vertical, If the system is in equilibrium, calculate the tension in one of the two wires. (Take $g = 10ms^{-2}$)

TRIANGLE OF FORCES

If three coplanar forces act on a body in such a way that the system is in equilibrium, then the forces can be represented in magnitude and direction by the sides of a triangle taken in order

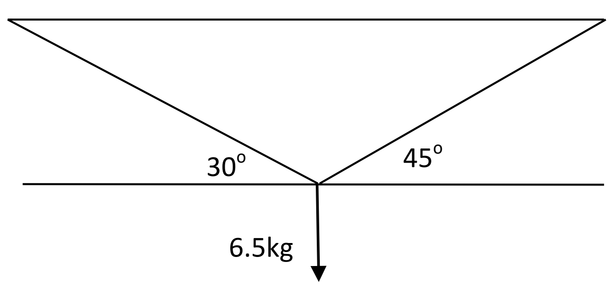


The triangle representing the three coplanar forces is called a triangle of forces.

Example:

1.A body of mass 6.5kg is supported by two strings, One of the stings is inclined at an angle of 30^0 and the other 40^0 to the horizontal . Find the tension in each strings, if the system is in equilibrium (take g = $10ms^{-2}$)

Solution



Using Sine rule;

$\underline{T1} = \underline{65}$

Sin 50° Sin 70°

$\underline{T1} = \underline{65 \times \sin 50^o}$

Sin 50° Sin 70

$T_1 = \dfrac{65 \times 0.766}{0.9397}$

$T_1 = 52.98N$

Similarly:

$\dfrac{T_2}{\sin 60^0} = \dfrac{W}{\sin 70^0}$

$T_2, = \dfrac{65 \times \sin 60^0}{\sin 70^0} = \dfrac{65 \times 0.8660}{0.9397}$

$T_2 = 59.9N$

EVALUATION

A body of mass s10kg is suspended by means of two light inextensible strings. AP and BP which are inclined at angles 60^0 and 30^0 respectively to the downward vertical. If T_1 and T_2 are the magnitude of the tension AP and BP respectively, calculate the values of T_1 and T_2.

LAMI'S THEOREM

This theorem states that if three forces acting at a point are in equilibrium, then each force is proportional to the sine of the angle between the liens of action of the other two forces.

Consider the forces F_1, F_2 and F_3 below

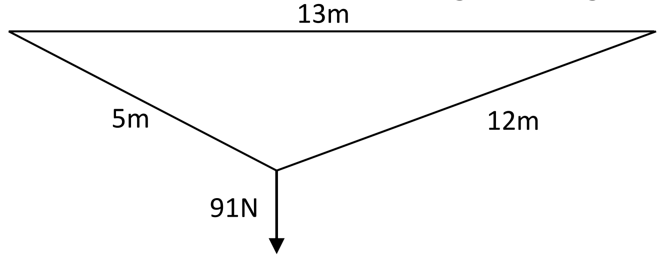
By Lami's theorem: F_1 ɑ Sin β F_2 ɑ sin γ F_3 ɑ sin θ

Since the forces are proportional to the sine of the angle: then $\frac{F1}{Sin\beta}=\frac{F2}{Sin\gamma} = \frac{F3}{Sin\theta}$

Example:

1. A body of weight 91N is suspended by two inelastic string 5m and 12 m long attached to two points on the same horizontal level, whose distance apart is 13m. Using Lami's theorem or otherwise, find the tension along the strings.

Solution



s

Le the tension in the strings be T_1nd T_2 and x, B be the angles made with the horizontal.

$AB^2 = OA^2 + OB^2 = 5^2 + 12^2$

$= 25 + 144 = 169$

but AB = 13

$AB^2 = 13^2 = 169$

$\therefore OA^2 + OB^2 = AB^2$

hence $\triangle$ AOB is a right angle triangle

$\therefore$ Sin B = $^5/_{13}$, sin x = $^{12}/_{13}$

B = $\sin^{-1} 0.3845$ Ɵ = $\sin^{-1} 12/13$

B = 22.6^0 = 0.9231 = 67.4^0

Using Lami's theorem:

$$\frac{T1}{\text{Sin } 157.4} = \frac{W}{\sin 90^0}$$

$$T_1 = \frac{91 \times \sin 157.4^0}{\text{Sin } 90^0} = \frac{91 \times 0.3843}{1}$$

$T_1 = 34.97N$

$$\frac{T_2}{\text{Sin } 112.6} = \frac{W}{\sin 90^0} \qquad T_2 = \frac{91 \times \sin 112.6}{\text{Sin } 90^0}$$

$$T_2 = \frac{91 \times 0.9232}{1}$$

$T_2 = 84.01N.$

EVALUATION

A particle of mass 10kg is connected by two strings of length 3m and 4m to two points on the same horizontal level and 5m apart, find the tension in the strings.

GENERAL EVALUATION

1. A particle of mass 98kg is suspended by two light inelastic strings of length 9m and 12m from two fixed point P and which are 15m apart. Calculate (i) the angles made by the strings with the upward vertical (ii) the tension in the strings.
2. The ends P and Q of an inextensible string 17m long are attached to two fixed points 13m apart on the same horizontal level. A body of mass 20kg is suspended from a point C on the string 5m from P. Calculate (a) the angle which each part of the string makes with the horizontal (b) the tension in each part of the string.

WEEKEND ASSIGNMENT

Two forces (8N, 030^0) and (10N, 120^0) act on a body; find the magnitude of the force that would be applied to keep the system in equilibrium.

In a community, 10% of the people tested positive to the HIV virus. If 6 persons from the community

are selected at random, one after the other with replacement, calculate correct to four decimal places,

the probability that (i) exactly 5 (ii) none (iii) at most 2, tested positive to the virus.

WEEK 9 STATICS – CONTINUATION

Definition of Moment of a Force

Principles of Moments

Application of the Principle in Solving Problems

Definition

Moments of a Force: The moment of a force about a reference point is defined as the product of the force and the force arm. Moment of a force is a vector quantity and its units is Nm.

The direction of movement or sense of movement about the point is duly considered.

The object can move in clockwise moment and ant-clockwise moments about the given point.

Suppose we have an object acted upon by two forces, F1 and F_2 in the opposite direction, then

$$|M_1| = |F_1| \times di \qquad |M_2| = |F_2| \times d_2$$

Where M_1 = Magnitude of F_1 $\qquad$ M_2 = Magnitude of F_2

$d1$ = Force arm of d_1 d_2 = Force arm of d_2

PRINCIPLE OF MOMENTS

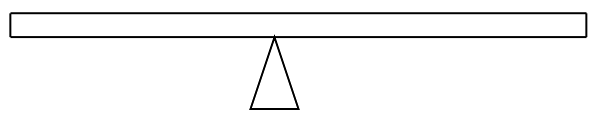
1. When a system of coplanar forces is in equilibrium, then the sum of the clockwise moment is equal to the sum of the anti-clockwise moments about the same point in the plane.

2. When the system is not in equilibrium, then the resultant of two coplanar forces F_1 and F_2 denote by R is represented by the relationship below:

$$M_1 + M_2 + M_R$$

Where M_1 = Moment of F1 M_2 = Moments of F_2 M_R = Moments of T.

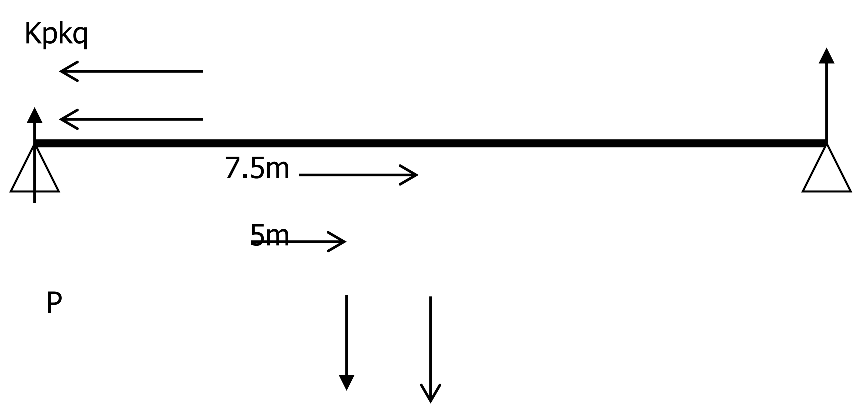
Centre of Gravity: The centre of gravity of a uniform plank or rod is the midpoint of the plant or rod



Examples

1. A uniform rod PQ is 15m long and has mass 20kg. The rod rests on two supports at P and Q. An object of mass 5kg is suspended at a point R on the rod 5m from the end P. Calculate the reaction of the supports P and Q (take $g = 10ms^{-2}$)

Solution



Q

5kg 20kg

Let the reaction at P be Kp and at Q beK_q

NB: The weight of the rod acts downward through the midpoint of the rod.

Moments about the point P, $M_R = M_1 + M_2$

But $M_1 = F_1 \times d_1$ and $F = Mg$.

$$Kq \times 15 = (5 \times 10) \times 5 + (20 \times 10) \times 7.5$$

$$15kq = 50 \times 5 + 200 \times 7.5$$

$$K_q = \frac{250 + 1500}{15.}$$

$$Kq = 116.\ 7N.$$

Moment about the point Q,

$$Kp \times 15 = (5 \times 10) \times 10 + (20 \times 10) \times 7.5.$$

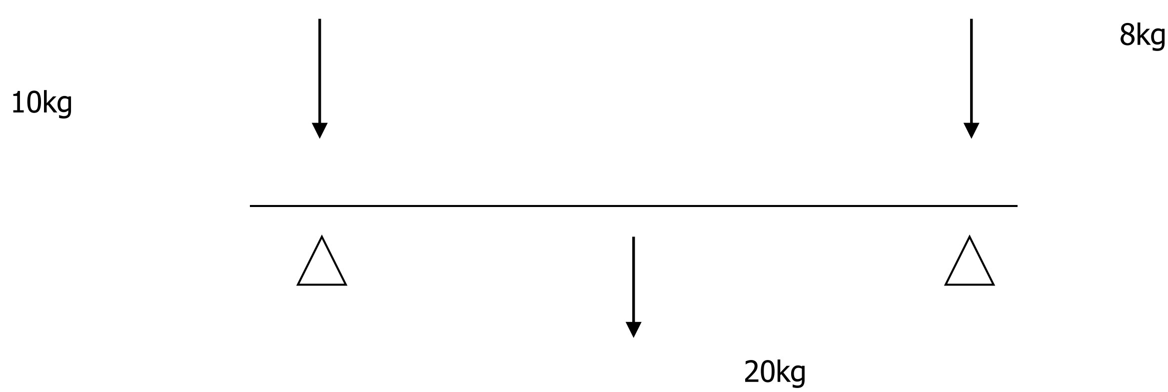
$$15Kp = 500 + 1500$$

$$Kp = \frac{2000}{15.}$$

$$Kp = 133.3N$$

1. A uniform beam AB of length 6m and mass 20kg rests on support P and Q placed 1m from each end of the beam. Masses of 10kg and 8kg are placed at A and B respectively. Calculate the reactions at P and Q ($g = 9.8ms^{-2}$)

Solution



Let reaction at point P be Rp and at point Q be Rq.

:. Moment about the point Q

Rp x 4 = (10 x 9.8) x 5 + (20 x 9.8) x 2 – (8 x 9.8 x 1)

4Rp = 490 + 392 - 78.4

Rp = <u>803.6</u>

4. Rp = 200.9N

Moment about the point P:

Rq x 4 = (8 x 9.8) x 5 + (20 x 9.8) x 2 - (10 x 9.8) x 1

4Rq = 392 + 392 – 98

Rq = <u>689</u>

4

Rq = 171.5N

EVALUATION

A uniform rod PQ, is 20 m long and weighs 80N, has weights 20 N and 50N suspended at P and Q respectively. Find the distance from P where the rod must be supported so that it will rest horizontally.

GENERAL EVALUATION

A uniform rod PQ of length 10 m and mass 2kg rest on two supports at x and y. If $|PX| = 2m$ and $|QY| = 1m$, find the reaction of X. (Take $g = 10ms^{-2}$)

WEEKEND ASSIGNMENT

A uniform beam PQ of length 100cm and of weight 35N lies on a support 40cm from the end P. Weights 54N and W are attached to the ends P and Q respectively to keep the beam in equilibrium. Find the value of W, to the nearest whole number.

FRICTION

Basic Concept

Coefficient of Friction

Forces Acting on a Body

Basic Concept: When two bodies are in contact, each one is exerting a force on the other. Therefore, friction can be defined as the force which tends to oppose the relative sliding motion of two surfaces in contact. Frictional force is the opposing force between two forces in contact.

The direction of friction is opposite to the direction in which the motion will occur. The frictional force for any two surfaces in contact has a value given by"

$F = UR$

Where R is the normal reaction between the bodies and obtained as; $R = m \times g$

U = Coefficient of friction and the value depends only on the nature of the surfaces in contact.

Examples

1: A mass of 4kg rests on a rough horizontal table, with U = 0.4. Find the least force sufficient to move the mass: (take $g = 10ms^{-2}$)

Solution

Sufficient force; $F = UR$.

$R = m \times g = 4 \times 10 = 40$

$F = 0.4 \times 40$

$F = 16N$

2. If a force of 10N is just sufficient to move a mass of 2kg resting on a rough horizontal table, find the coefficient of friction ($g = 10ms^{-2}$)

Solution

$F = 10$ N , $m = 2$kg.

$F = UR$

$10 = U \times 2 \times 10$

$10 = 20U$

$u = 10/20$

$U = 0.5$

EVALUATION

A body of mass 8kg rests on a horizontal surface. If the coefficient of friction between the body and the horizontal surface is 0.65, calculate the minimum horizontal force enough to just move the body.

FORCES ACTING ON A BODY PLACED ON A ROUGH INCLINED PLANE

Rough Inclined Plane: There are two basic forces acting on an object placed on an inclined plane; the applied forces P and the frictional force F P= Mg Sin Θ

F = Mg SinΘ

R = Mg Cos Θ

Recal F = *U*R

:. F = *U*MgCos Θ where F is the limiting

Friction:

The least force required in P to make the body move up the inclined plane r is P = UR + Mg Sin Θ

:. P = F + MgSinΘ

P = UR + MgSinΘ

The least force required to make the body slide down the plane is thus

P + f = mgSin Θ

P = mg Sin Θ- *U*r.

Where m = mass and Θ= angle of friction.

The value of the coefficient of fricton from F = *U*R and

MgSinΘ = *U*MgCos Θ is defined by

$$U = \frac{mg \sin \Theta}{Mg \cos \Theta}$$

:. U = tan Θ

Examples

1. A block is placed on an inclined plane at an angle of 30^{o} to the horizontal and just remains at rest. Find the coefficient of friction.

Solution

$\Theta = 30^o$, $U = \tan \Theta$

$U = \tan 30^o$

$U = 1/\sqrt{3}$ or 0.5774.

2. A body of mass 4.5kg rests on a smooth plane inclined at an angle of 47^o to the horizontal. Calculate the magnitude of the force P parallel to the plane just enough to prevent the body from sliding down the plane
($g = 9.8ms^{-2}$).

Solution

$P = mg \sin \Theta$

$P = 4.5 \times 9.8 \times \sin 47^o = 44.1 \times 0.7314$.

$P = 32.25N$.

EVALUATION

1. A particle of mass 25kg slides down a rough plane inclined at angle 30^o to the horizontal. If the coefficient of friction is 0.2, find, in ms^{-2} the acceleration of the particle correct to 3 significant figures (take $g = 10ms^{-2}$)

2. A uniform ladder of length 6m and mass 30kg rests with one end against a rough vertical wall and the other end on a rough horizontal ground. The coefficient of friction at each point of contact is 0.3. If the ladder is on the point of slipping, calculate the (i) normal reaction of the ground (ii) frictional force at the wall (iii) angle of inclination of the ladder to the ground. (Take $g = 10ms^{-2}$)

WEEKEND ASSIGNMENT

1. A body is in limiting equilibrium on a plane inclined at an angle to the horizontal. if Cos = 0.8, calculate the coefficient of friction.

2. A big stone is of mass 13kg. A boy whose weight is 59N sits on the stone. Find the minimum horizontal force P required to move the stone on the ground. If the coefficient of friction is 0.25 ($g = 9.8ms^{-2}$).

3. A mass of 8kg tests on a rough table with $U = 0.6$. Find the least force which will make the mass move ($g = 10ms^{-2}$).

4. A mass of 10kg slides down a rough plane inclined at Θ to the horizontal where $\sin \Theta = 0.6$. if $U = 0.3$, find the acceleration of the box.

5. A body can just rest in equilibrium on a slope inclined at Θ to the horizontal where $\sin\Theta = {}^5/_{13}$, find U