

CLASS: SS 2
SUBJECT: PHYSICS

FIRST TERM: E-LEARNING NOTES

FIRST TERM SCHEME OF WORK

WEEK: TOPIC

1. Revision of last year's work.
2. Position, distance and displacement: (a) concept of position and position coordinate (b) frame of reference
3. Vectors; (a) concept of vectors (b) vector representation (c) addition of vectors (d) resolution of vectors (c) concept of resultant velocity using a vector representation.
4. Equation of uniformly accelerated motion; (a) Revision on velocity-time graph (b) application and interpretation of equation of motion in simple problems.
5. Projectile: (a) concept of projectiles (b) simple problems involving range, height and time of flight
6. Equilibrium of forces; (a) resultant and equilibrant forces (b) parallel forces (c) moment of a force (torque) (d) centre of gravity and equilibrium (e) equilibrium of forces in a liquids
7. MID-TERM BREAK
8. Equilibrium of Bodies in Liquids: Concept of upthrust, Archimedes' Principle, Density & Relative Density, law of floatation and calculations.
9. Simple Harmonic Motion (SHM): (a) definition of simple Harmonic Motion (b) speed and acceleration of SHM (c) period, frequency and amplitude of SHM (d) energy in SHM (e) force vibration in SHM.
10. Revision
11. examination

REFERENCES:

1. new school Physics by MW Anyakoha
2. New system PHYSICS for senior secondary schools. Dr. Charles Chew.
3. Comprehensive Certificate Physics by Olumuyiwa Awe
4. Senior School Physics BY PN Okeke, SF Akande
5. STAN Physics.

WEEK ONE:

DATE.....

TOPIC: REVISION OF LAST YEAR'S WORK.

CONTENT:

The educator should make a good revision of the last term's/year's work especially on areas the students are having problems.

GENERAL EVALUATION:

Educator should construct questions based on previous term's/year's work to ascertain the level of recalling.

WEEKEND ASSIGNMENT:

Educator should give the students questions based on the previous term's/year's work and also prepare the students' mind for week 2.

READING ASSIGNMENT

Read up the topic: "Position Distance and Displacement" in the following text books.

- i. Senior Secondary School Physics by P.N. Okeke et al.
- ii. New School Physics for Senior Secondary Schools by Anyakoha, M.W.

WEEK TWO. DATE.....

TOPIC: Position, Distance and Displacement

CONTENT:

- Concept of position

- Location of point on a Cartesian plane
- Distance
- Estimation of distance between two point on a Cartesian plane
- Displacement
- Difference between distance and displacement
- Frame of reference

PERIOD ONE: POSITION

This is the **location** of a point/object with respect to a **reference point**. The position of a point in space is defined in terms of the distance of the point from the reference point (which is sometimes called ORIGIN). In physics, the position of an object in space is represented in a coordinate system. There are three main types of coordinate system for representing the position of an object in space:

1. Cartesian coordinate system
2. Spherical coordinate system
3. Cylindrical coordinate system

Of all these, the Cartesian coordinate system is the most commonly used.

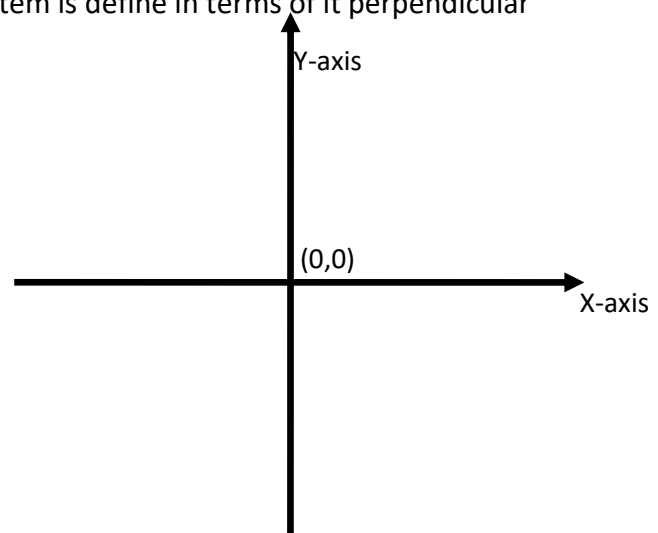
CARTESIAN COORDINATE SYSTEM:

This is also called the **rectangular** coordinate system. This consists of two (or three) mutually perpendicular axes. The Cartesian plane in two dimensions consists of two mutually perpendicular axes:

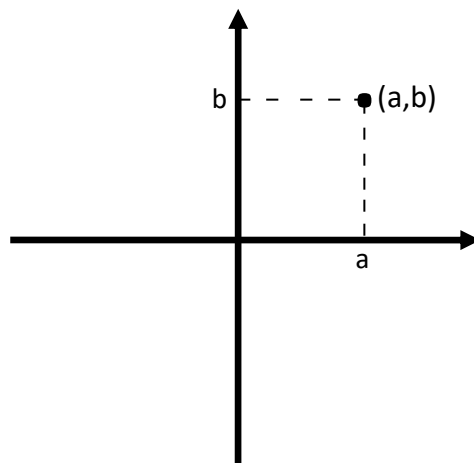
- the horizontal axis (also called the **X axis** or the **abscissa**)
- the vertical axis (also called the **Y axis** or the **ordinate**).

The position of a point in this coordinate system is define in terms of it perpendicular distance from these axes.

(0,0) is the origin.

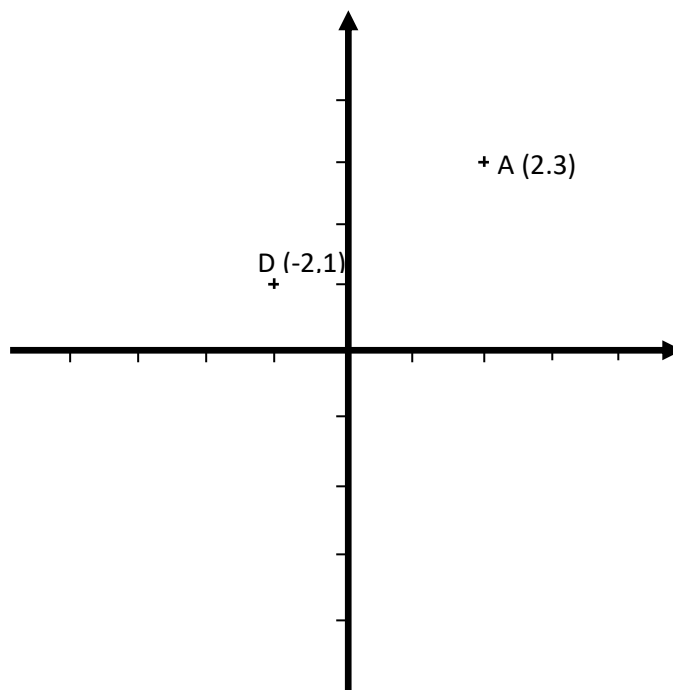


For instance the position of a point P define as (a,b) is represented as shown.



This is similar to the location of point on a graph sheet when plotting points.

CLASS ACTIVITY: locate the following point on the graph sheet below. A(2,3) B(1,-1) C(2,-3)
D(-2,1) E(0, 2)



Locate the remaining points.

EVALUATION: On a graph sheet, locate the following points

1. (2, -5)
2. (-3, -2)
3. (2.6, -3.4)
4. (-5.1, 6.3)
5. (2.76, 1.92)

PERIOD TWO :DISTANCE.

This can be defined as the actual length measured along the path moved by an object.

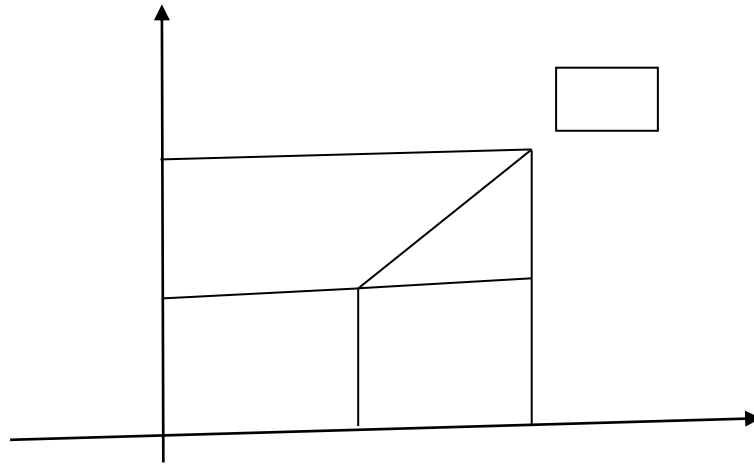
Distance is a scalar quantity and its S.I unit is metre (m). If an object moved along a straight line, the distance moved is the length of the straight line. If the path is a curve, then the distance moved is the length of the curve.

DISPLACEMENT:

This is the distance moved in a specified direction. Displacement is a vector quantity and its S.I unit is metre.

ESTIMATION OF DISPLACEMENT BETWEEN TWO POINTS ON THE CARTESIAN PLANE

Consider the point P and Q on a Cartesian plane. If the coordinate of P and Q is given as: $P(x_1, y_1)$ and $Q(x_2, y_2)$, then the displacement between P and Q on the Cartesian plane is given as



$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Example: Calculate the distance between the two points: $P(4, 2)$ and $Q(1, 6)$

Solution:	$P(x_1, y_1)$	$Q(x_2, y_2)$
	$P(4, 2)$	$Q(1, 6)$
	$x_1 = 4, y_1 = 2$	$x_2 = 1, y_2 = 6$

$$D = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$D = \sqrt{(1 - 4)^2 + (6 - 2)^2}$$

$$D = \sqrt{(-3)^2 + (4)^2}$$

$$D = \sqrt{9 + 16} = \sqrt{25} = 5 \text{ units}$$

DISPLACEMENT BETWEEN TWO POINTS ON THE CARTESIAN PLANE

Consider the points P and Q on a Cartesian plane. If their coordinates are: $P(x_1, y_1, z_1)$, $Q(x_2, y_2, z_2)$, then the distance between P and Q on the Cartesian plane is given as

$$D = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

Class Activity: Calculate the distance between the points $P(2, 0, 5)$ and $Q(3, -2, 1)$

Soln:

$$P(2, 0, 5) = (x_1, y_1, z_1)$$

$$Q(3, -2, 1) = (x_2, y_2, z_2)$$

$$D = \sqrt{[(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2]}$$

$$D = \sqrt{[(3 - 2)^2 + (-2 - 0)^2 + (1 - 5)^2]}$$

$$D = 4.58 \text{ units}$$

Differences between distance and displacement

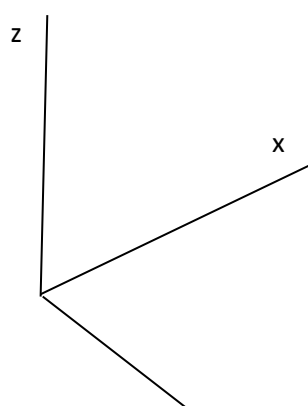
Distance	Displacement
1. It is the actual length of the path moved by an object.	It is the distance moved in a specified direction.
2. It is a scalar quantity	It is a vector quantity

EVALUATION;

1. Calculate the distance between the following set of points.
 - (i) (2, 5) and (-4, -3)
 - (ii) (8, 7) and (0, -8)
 - (iii) (6, 6) and (-6, 1)
 - (iv) (-4, 14) and (8, 6)
2. The distance between the points A(p, -2) and B(3, -8) is 10 units. What is the value of p?

PERIOD THREE: frame of reference

This is a set of axes used to specify the position of object in space at any instant of time. For practical purposes, the frame of reference of the earth is taken to be at rest (i.e. an *inertia frame of reference*). However, this is never so. In two dimensional continuums, the frame of reference consists of two axes.



This is a three dimensional frame of reference to specify the position of an object at any time in space.

In four dimensional continuums, the time coordinate is added to the space coordinate (x, y, z). Hence for three dimensional frames of reference position is defined as (x,y,z). But for four dimensional frame of reference, position is define as (x,y,z,t) – (space-time)

When an event in a frame of reference is observed in two frame of reference moving relatively with respect to each other, their observations will be different. This leads to the concept of **relativity**. (see *Einstein theory of special relativity*)

However, **all frames of reference moving at a constant velocity with respect to each other are equivalent**. All frames of reference at rest or moving with uniform velocity are called **Galilean frames** and that are equivalent for describing the dynamics of moving bodies.

EVALUATION

1. What is an inertia frame of reference?
2. What is a Galilean frame of reference?

GENERAL EVALUATION

1. The following are types of coordinate system except (a) rectangular coordinate system **(b)** cubical coordinate system (c) cylindrical coordinate system (d) spherical coordinate system
2. Another name for the horizontal axis of a Cartesian coordinate system is (a) Y-axis (b) ordinate **(c)** abscissa (d) coordinate
3. An ant on a graph page moved starting from the origin to another point (-6, 8). What is the displacement of the ant? (a) 4units (b) 7units (c) 9units (d) 10units
4. A rat on a horizontal frame of reference moved from (13, 7) metres to another point (x, 0) metres. For what value of x will the displacement of the rat be 25m? (a) 16 (b) 21 (c) 37 (d) 43
5. ---- is the distance moved in a specified direction. (a) vector (b) displacement (c) distance (d) scalar
6. A body moving with uniform acceleration **a** is represented by points (10, 30) and (25, 65) on a velocity-time graph. Calculate the magnitude of **a**. (a) 0.47ms^{-2} (b) 0.50ms^{-2} (c) 0.60ms^{-2} (d) 1.67ms^{-2} (e) 2.33ms^{-2}

Essay

1. Differentiate between distance and displacement
2. Sketch a Cartesian plane and locate the following points on it.

- (i) (-3, 4)
- (ii) (5, -2)
- (iii) (4, 0)
- (iv) (1.2, -4.6)
- (v) (5.72, 3.31)

WEEKEND ASSIGNMENT: what is the difference between an inertia frame of reference and a non-inertia frame of reference?

READING ASSIGNMENT: read pages 111-116 of the New School Physics by MW Anyakoha and answer question 7 and 8 on page 121

WEEK 3:

DATE -----

TOPIC: VECTORS

CONTENT:

- Concept and examples of scalars
- Concept of vectors
- Distinction & similarities between scalars & vectors

- **Examples of vectors**
- **Representation of vectors**
- **Addition of vectors**
- **Resolution of vectors**

Sub-Topic 1: CONCEPT OF SCALARS

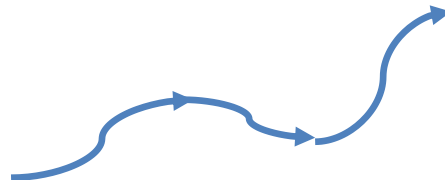
Scalars are physical quantities that have magnitude but no direction. That is, scalar has value and unit but no direction. E.g, 10km. This 10km could be in any direction since there is no actual direction. The '10' is the value- the magnitude. Therefore, just 10km is a scalar quantity. Scalar quantities are always not directional.

Scalar quantities unlike vectors have only magnitude. Example; length, area, volume, temperature, work, energy, power, mechanical advantage, velocity ratio, efficiency, surface tension,

Other examples of scalar quantities include:

- Speed
- Time
- Density
- Mass
- Distance, etc.

10km



Scalars are non directional physical quantities.

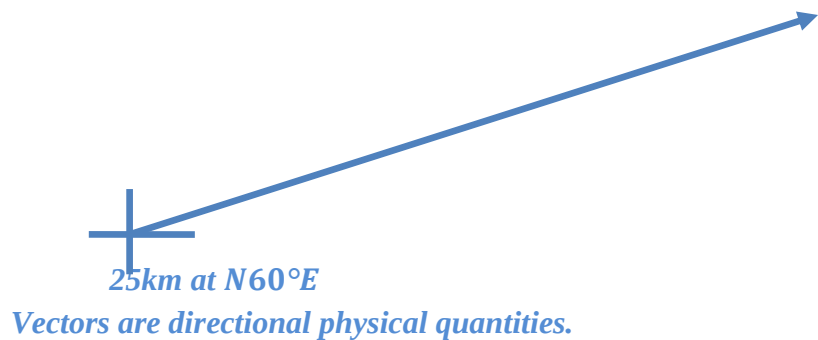
CONCEPT OF VECTORS

Vectors are physical quantities that have both magnitude and direction. This means that vectors quantities have values and are always directional. E.g, 10km due North. Here, the value, which is the magnitude, is '10' while the direction is North.

Examples of vector quantities include: pressure, friction, tension, electric field intensity, magnetic field intensity, moment of forces, torque, upthrust.

Other examples of vector quantities are:

- 25km at $N60^\circ E$
- Displacement
- Force
- Acceleration
- Momentum
- Impulse
- Velocity
- Weight, etc.



Evaluation:

1. What are vector quantities?
2. List five examples of each of the two types of the physical quantities.

Sub-Topic 2: DISTINCTION BETWEEN SCALARS AND VECTORS.

S/N	SCALARS	VECTORS
1.	Scalars are non directional physical quantities.	Vectors are directional physical quantities.
2.	Always directed towards different directions.	Always directed towards a particular direction.
3.	E.g, 100km	100km due east.
4.	E.g, mass	Weight

Their similarities include:

- They are both physical quantities;
- They both have values, which are the magnitudes.

Evaluation:

1. State three differences between scalars and vectors.
2. State the similarities between them.

GENERAL EVALUATION:

1. What is the difference between 20km and 20km due South?
2. How would you differentiate a scalar from a vector quantity?
3. List five examples of scalar quantities.
4. Mention five physical quantities you consider as vectors and why.

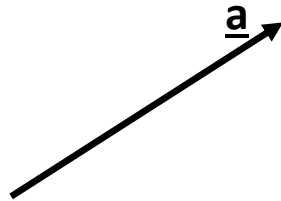
Types of vectors

1. Position vectors; these are vectors whose starting point is fixed to a position
2. Free vectors; these are vector whose starting point could be anywhere in space.
3. Unit vector; this is a vector whose magnitude is one. It is often represented as $\hat{a}$.
4. Orthogonal vectors; these are vectors whose lines of action are mutually perpendicular to each other
5. Collinear vectors; these are vector whose lines of action are parallel to one another.

6. Coplanar vectors; these are vectors whose lines of action lie on the same plane.
7. Resultant vector; this is a single vector that has the same effect as a system of vectors.
8. Null vector: this is a vector whose magnitude is zero.

Representation of vectors

Vectors can be represented by a **directed line segment** whose length is proportional to the magnitude of the vector and its direction is pointing in the direction of action of the vector.



Vectors are represented with bold face letters $\mathbf{a}$, $\underline{\mathbf{a}}$, or $\hat{\mathbf{a}}$.

EVALUATION:

1. A measurable quantity that has both magnitude and direction is called ---- (a) vector (b) scalar (c) displacement (d) distance
2. The following are examples of vectors except ---- (a) moment (b) pressure in gas (c) tension (d) viscosity
3. A vector whose magnitude is one is called ---- (a) collinear vector (b) orthogonal vector (c) unit vector (d) free vector
4. A set of vectors whose line of action lies on the same plane is called ---- (a) collinear vectors (b) concurrent vectors (c) coplanar vectors (d) coordinate vectors
5. Which of the following groups of quantities is NOT all vectors? (a) (a) momentum, velocity, force (b) acceleration, force, momentum (c) momentum, kinetic energy, force (d) magnetic field, acceleration, displacement

PERIOD TWO: Addition of vector

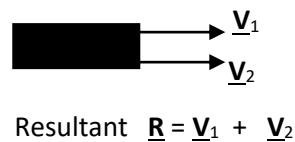
The addition of two or more vectors produces a single vector called the resultant vector.

A resultant is a single vector which has the same effect as a system of vectors put together. Equilibrant is the vector that will bring a system of vectors to equilibrium when added to the system. It has the same magnitude as that of the system but acting in the opposite direction to the resultant.

Consider two vectors $\underline{\mathbf{a}}$ and $\underline{\mathbf{b}}$, the addition of these vectors can be obtained by joining the head of one to the tail of the previous one. The resultant is the vector that joins the beginning to the end.



Case 1. Parallel vectors acting in the same direction



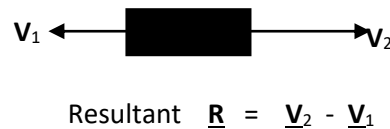
For two parallel vectors acting in the same direction, the angle between the vectors is **ZERO**

Example 1: Three men pushed a car out of a muddy ground by applying the following forces 450N, 600N and 920N. What is the resultant force on the car?

$$R = V_1 + V_2 + V_3$$

$$R = 450 + 600 + 920 = 1970N$$

Case 2. Parallel vectors acting in opposite direction



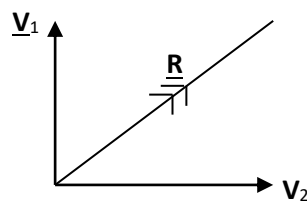
For two parallel vectors acting in opposite direction, the angle between the vectors is **180°**

Example 2: during a tug of war game, team A pull in the positive x direction with a force of 900N and team B pull in the negative X – direction with a force of 1200N. what is the resultant of the train?

$$V_1 = 900N \quad V_2 = 1200N$$

$$R = V_2 - V_1 = 1200 - 900 = 300N \text{ in the negative x-direction.}$$

Case 3. Two perpendicular vectors acting at a point.



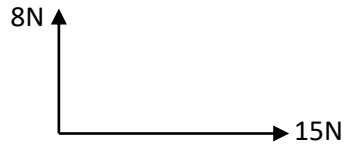
$$\text{Resultant } R = \sqrt{V_1^2 + V_2^2}$$

The angle made by the resultant with the direction of $\underline{V}_2$ is given as

$$\theta = \tan^{-1} \left(\frac{V_1}{V_2} \right)$$

Example 3: two force 8N and 15N acting along the vertical and the horizontal axis respectively acts on a body of mass 3kg. What is the acceleration of the body?

Solution:



$$\text{Resultant } R = \sqrt{V_1^2 + V_2^2}$$

$$R = \sqrt{8^2 + 15^2}$$

$$R = \sqrt{64 + 225}$$

$$R = \sqrt{289} = 17N$$

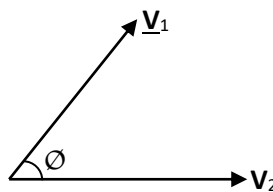
Force = mass x acceleration

$$F = ma \quad F = R = 17N \quad m = 3\text{kg} \quad a = ?$$

$$17 = 3 \times a$$

$$a = \frac{17}{3} = 5.7 \text{ ms}^{-2}$$

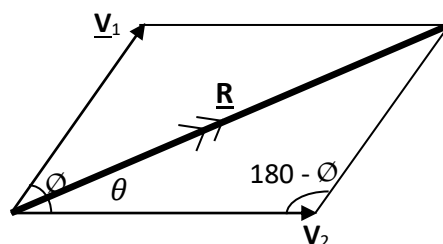
Case 4. Two vector acting at a point and at angle to each other.



This case can be solved by using the parallelogram law of vectors.

Parallelogram law of vectors state that:

Parallelogram law of vectors state that when two vectors **acting at a point** are represented in magnitude and direction by the **adjacent sides** of a parallelogram, the resultant of the two vectors can also be represented in magnitude and direction by the **diagonal** of the parallelogram drawn from the **common point** of the two vectors.



$$R^2 = V_1^2 + V_2^2 - 2V_1V_2 \cos(180 - \phi)$$

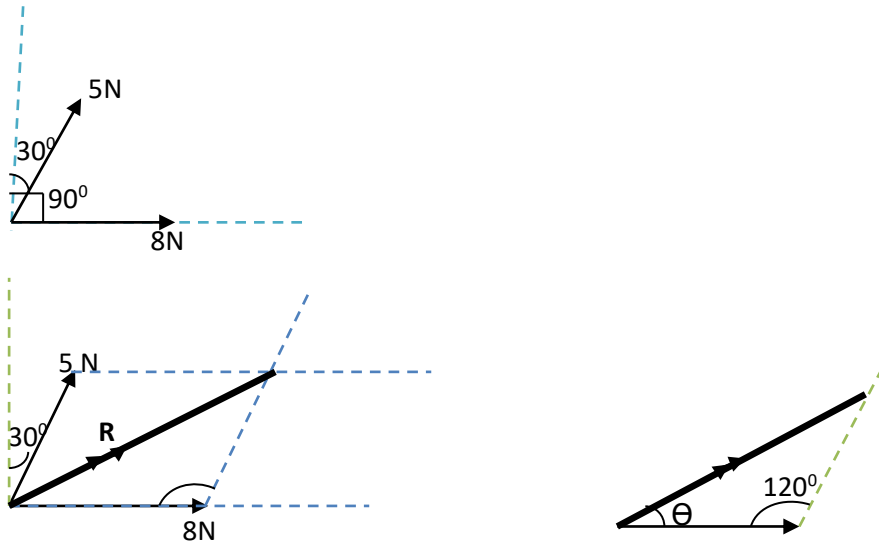
ϕ is the angle between the two vectors. The direction the resultant force R made with V_2 (i.e. θ) can be obtained using the sin rule. This is given as

$$V_1 \sin(180 - \phi) = R \sin \theta$$

where θ is the angle the vector made with the direction of V_2

Example: two forces F_1 and F_2 act on a particle. F_1 has magnitude 5N and in direction 030° , and F_2 has a magnitude of 8N and in the direction 090° . Find the magnitude and direction of the resultant.

Solution:



The angle between the two forces is $090^\circ - 030^\circ = 60^\circ$

$$R^2 = F_1^2 + F_2^2 - 2F_1F_2 \cos \phi$$

$$R^2 = 5^2 + 8^2 - 2(5)(8) \cos(180 - 60)$$

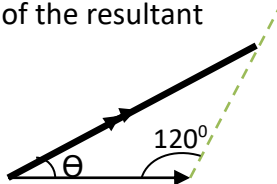
$$R^2 = 25 + 64 - 80 \times \cos 120$$

$$R^2 = 89 - 80 (-0.500)$$

$$R^2 = 89 + 40$$

$$R = \sqrt{129} = 11.4N$$

(b) direction of the resultant



don't forget

- 120° is obtained from $(180^\circ - 60^\circ)$ in the diagram
- we use V_1 because we are looking for the angle between R and V_2 .

$$F_1 \sin(180 - \phi) = R \sin \theta$$

$$5 \sin(120) = 11.4 \sin \theta$$

$$\sin \theta = \frac{5 \times 0.866}{11.4} = \frac{4.33}{11.4}$$

$$\theta = \sin^{-1} 0.3798 = 22.3^\circ$$

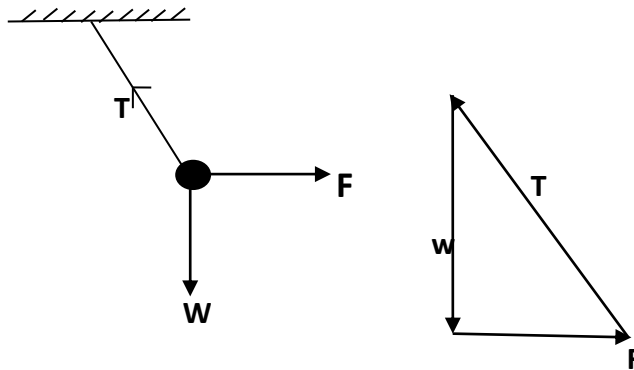
So the direction of the resultant in three digits ($90 - 22$) = 068°

EXPERIMENT 1:

- Educator should carry out an experiment to verify the parallelogram law of vectors
- using the force board

CASE 5. Three vectors acting at a point and in equilibrium.

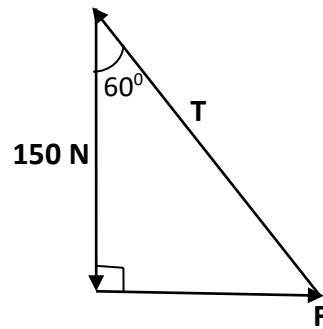
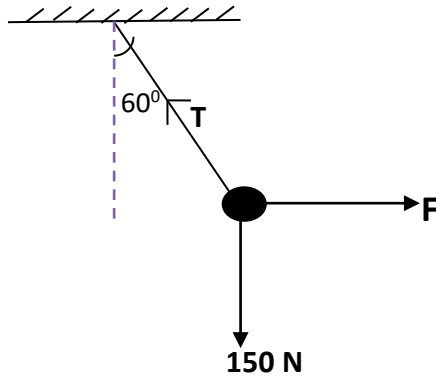
Consider a metal ball suspended from a ceiling by a string. If is pulled by an horizontal force as shown below, the triangular law of vector may be applied as shown below.



Triangular law of vectors states state that when three vectors acting act a point are in equilibrium, the vectors can be represented in magnitude and direction by the adjacent sides of a triangle by joining the head of one vector to the tail of the previous one.

Example: a 15 kg mass suspended from a ceiling is pulled asides with a horizontal force, F, as shown in the diagram above. Calculate the value of the tension T and the horizontal force ($g= 10\text{m/s}^2$)

Solution:



Using the trigonometric ratio;

We applied the Lami's theorem to get the triangle

$$\cos\theta = \frac{150}{T}$$

$$\cos 60 = \frac{150}{T}$$

$$T = \frac{150}{\cos 60}$$

$$T = \frac{150}{0.500}$$

$$T = 300\text{ N}$$

EXPERIMENT 2:

- An experiment to verify the Lami's theorem using the force board.

N.B. note that the **resultant** of a system in **equilibrium** is **ZERO**.

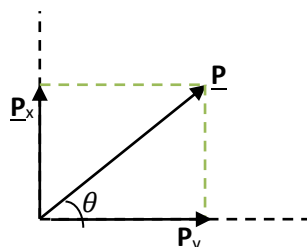
The triangular law of vectors is also called the **Lami's theorem**

EVALUATION

1. The angle between two parallel vectors acting in opposite direction is ---- (a) 0° (b) 45° (c) 90° (d) 180°
2. The resultant of a system of forces in equilibrium is ---- vector. (a) unit (b) free (c) orthogonal (d) null
3. What is the resultant of the forces 6N and 8N acting at an angle 60° to each other? (a) 9.0N (b) 10.1 N (c) 12.2N (d) 13.5N
4. Two vectors **a** and **b** act on a body. What will be the angel between a and b for the resultant to be maximum. (a) 0° (b) 45° (c) 90° (d) 180°

PERIOD THREE: Resolution of vectors (Vector components)

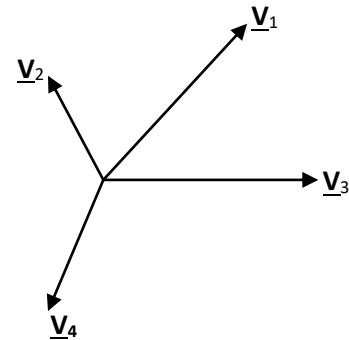
Any position vector can be resolved into two components which are perpendicular to each other. Consider the vector **P** acting at angle θ to the horizontal as shown below,



P_x and p_y are components of the vector p .

For a system which consist of several vectors, each vector in the system can be resolved into two components as shown above.

Consider a system of vectors as shown below,



If $\theta_1, \theta_2, \theta_3$, and θ_4 are the angles made the vectors V_1, V_2, V_3 and V_4 respectively, then the component of the resultant along the horizontal is given as:

$$R_x = \sum_{n=1}^4 V_n \cos \theta_n$$

$$R_x = V_1 \cos \theta_1 + V_2 \cos \theta_2 + V_3 \cos \theta_3 + V_4 \cos \theta_4$$

And the vertical component of the resultant vector is given as:

$$R_y = \sum_{n=1}^4 V_n \sin \theta_n$$

$$R_y = V_1 \sin \theta_1 + V_2 \sin \theta_2 + V_3 \sin \theta_3 + V_4 \sin \theta_4$$

Note: $\theta_1, \theta_2, \theta_3, \theta_4$ refers to the angles the vectors V_1, V_2, V_3 , and V_4 makes with the positive x direction

The magnitude of resultant R is given as:

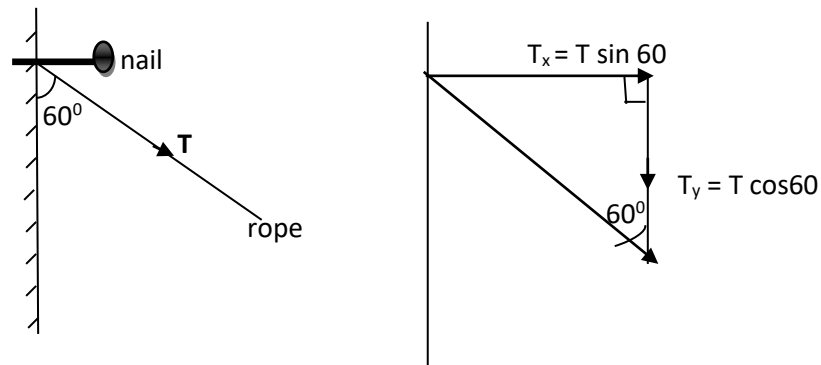
$$R = \sqrt{R_x^2 + R_y^2}$$

The direction of the resultant with respect to the positive x direction is given as

$$\theta = \tan^{-1} \left[\frac{R_y}{R_x} \right]$$

Example: A boy pulls a nail from the wall with a string tied to the nail. The string is inclined to the wall at angle 60° . If the tension in the string is 4N, what is the effective force used in pulling the nail?

Solution;



The tension has two components T_x and T_y . note that the value of T_x and T_y were obtained using the trigonometric ratio.

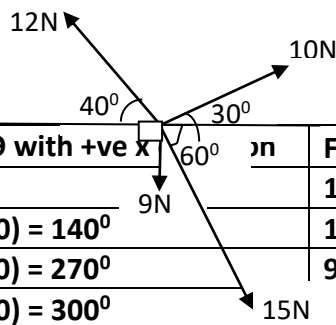
The component of T to extract the nail is T_x .

$$T_x = T \sin 60$$

$$T_x = 4 \times 0.866 = 3.464 \text{ N}$$

EXAMPLE: Four forces act at a point as shown below. Calculate the magnitude and direction of the resultant force.

Solution:



Vector F	F_i	Angles θ with +ve x	$F_x = F \cos \theta$	$F_y = F \sin \theta$
F_1	10N	30°	$10 \cos 30 = 8.66$	$10 \sin 30 = 5.00$
F_2	12N	$(180 - 40) = 140^\circ$	$12 \cos 140 = -9.192$	$12 \sin 140 = 7.713$
F_3	9N	$(180 + 90) = 270^\circ$	$9 \cos 270 = -0.000$	$9 \sin 270 = -9.00$
F_4	15N	$(360 - 60) = 300^\circ$	$15 \cos 300 = 7.50$	$15 \sin 300 = -12.99$
			$\sum F_x = 25.35$	$\sum F_y = -9.277$

OR

$$F_x = F_1 \cos \theta_1 + F_2 \cos \theta_2 + F_3 \cos \theta_3 + F_4 \cos \theta_4$$

$$F_x = 10 \cos 30 + 12 \cos 140 + 9 \cos 270 + 15 \cos 300$$

$$F_x = 10 \times 0.866 + 12 \times -0.766 + 9 \times 0 + 15 \times 0.500$$

$$F_x = 8.66 - 9.192 + 0 + 7.500 = 25.35 \text{ N}$$

Vertical components

$$F_y = F_1 \sin \theta_1 + F_2 \sin \theta_2 + F_3 \sin \theta_3 + F_4 \sin \theta_4$$

$$F_y = 10 \sin 30 + 12 \sin 140 + 9 \sin 270 + 15 \sin 300$$

$$F_y = 10 \times 0.500 + 12 \times 0.6428 + 9 \times -1 + 15 \times -0.866$$

$$F_y = 5.00 + 7.713 - 9 - 12.99 = -9.277$$

$$R = \sqrt{F_x^2 + F_y^2}$$

$$R = \sqrt{25.35^2 + (-9.277)^2}$$

$$R = \sqrt{642.6 + 86.06} = \sqrt{728.66}$$

$$R = 27N$$

Direction of the resultant force

$$\theta = \tan^{-1} \left[\frac{R_y}{R_x} \right]$$

$$\theta = \tan^{-1} \left[\frac{-9.277}{25.35} \right]$$

$$\theta = \tan^{-1} [-0.366]$$

$$\theta = 20.1^\circ$$

θ is negative. $\tan \theta$ is negative in the 2nd and the 4th quadrant. Looking at the geometry of the forces, R will be in the 4th quadrant.

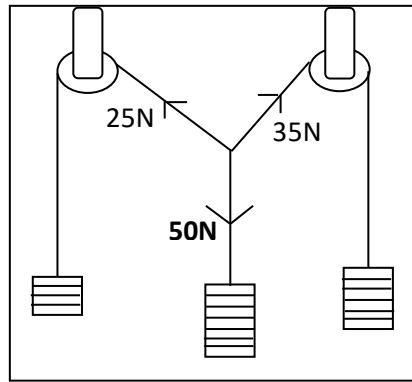
$$\therefore \theta = 360 - \theta = 360 - 20.1$$

$$\theta = 339.9^\circ$$

This is the angle made by the resultant with the positive x-axis.

EVALUATION.

1. The component of a force along the vertical and the horizontal axis is given as 24N and 7N respectively. What is the magnitude and direction of the resultant force?
2. The resultant of two forces 12N and 5N is 13N. What is the angle between the two forces? (a) 0° (b) 45° (c) 90° (d) 180°
3. Two uniform velocities are represented by $\underline{V}_1$ and $\underline{V}_2$. If the angle between them is θ . Where $0^\circ \leq \theta \leq 90^\circ$, the magnitude of their resultant is ----
 (a) $(V_1^2 + V_2^2 + 2V_1V_2 \sin \theta)^{1/2}$ (b) $(V_1^2 + V_2^2 + 2V_1V_2 \cos \theta)^{1/2}$
 (c) $(V_1^2 + V_2^2 - 2V_1V_2 \cos \theta)^{1/2}$ (d) $(V_1^2 + V_2^2 - 2V_1V_2 \sin \theta)^{-1/2}$
4. Below is the diagram of an experiment to determine the resultant of a system using a force board, calculate the angle between the 25N and the 35N.



5. The wind velocity is 30ms^{-1} , 30° north of West. Find the component in the north and West direction
6. A force of 15N acts in the positive x-direction. In what direction to the positive x-direction will a force of 20N be applied to give a resultant whose component along the x-direction is zero?

GENERAL EVALUATION:

1. A boy drags a heavy crate along the horizontal ground with a string inclined to the horizontal at 50° . if the tension in the string is 1500N, calculate the effective force pulling the crate along the ground.
2. A body is in equilibrium under the action three forces. One of the force is 6.0N acting due East and one is 3.0N in a direction 60° North of East. What is the magnitude and direction of the third force?
3. Two forces acting at a point makes angles of 25° and 65° respectively with their resultant which is of the magnitude 15N. find the magnitudes of the two component forces.
4. Differentiate between scalar and vector
5. The resultant of two forces acting on an object is maximum if the angle between them is (a) 180° (b) 90° (c) 45° (d) 0°

WEEKEND ASSIGNMENT 1:

1. 200 inches due east is an example of ---
 - A. Displacement
 - B. Speed
 - C. Acceleration
 - D. Distance
2. 20km due east is an example of ---
 - A. Scalar quantity
 - B. Vector quantity
 - C. Fundamental quantity
 - D. Basic quantity
3. 500 miles is an example of, ..

- A. Displacement
- B. Distance
- C. Force
- D. Speed

4. The following are examples of scalar quantities. Except ---

- A. Time
- B. Density
- C. Velocity
- D. Mass

5. One of the following is a vector quantity.

- A. Force
- B. Distance
- C. Mass
- D. Speed

Essay

1. Tabulate the below physical quantities into scalars and vectors.

Force, acceleration, speed, velocity, time, mass, weight, distance, momentum, displacement.

2. (a) What is a vector quantity? **(2mks)**

(b) Three vectors 3ms^{-1} N450W, 12ms^{-1} W and 5ms^{-1} S act at a point.

(i) Sketch a vector diagram to illustrate the given information. **(2mks)**

(ii) Calculate the resultant of the vectors. **(4mks)**

(c) In a laboratory experiment to determine the force constant of a spiral spring, the mass on the spring was varied and the corresponding extensions were measured and recorded as shown in the table below.

Mass m/g	Weight W/N	Extension e/cm
50.0		6.50
100.0		11.00
150.0		15.00
200.0		20.00
250.0		25.00

(i) Complete the table above (Take $g = 10\text{ms}^{-2}$) **(2mks)**

(ii) Plot a graph with weight, W, on the vertical axis and extension, e, on the horizontal axis. **(2mks)**

(iii) Using the graph, determine the force constant of the spring. **(2mks)**

(iv) Determine the natural length of the spring if its length was 38.0cm when loaded with 250g mass. **(1mk)**

(WAEC 2011 Q11)

READING ASSIGNMENT

Read up the topic: “Work, Energy and Power” in the following text books.

- i. Senior Secondary School Physics by P.N. Okeke et al.
- ii. New School Physics for Senior Secondary Schools by Anyakoha, M.W.

WEEKEND ASSIGNMENT 2:

1. Differentiate between a resultant of force and equilibrant
2. Describe an experiment to determine the resultant of a system of force using the force board.

READING ASSIGNMENT: Students should read pages 112-118 of New School Physics by MA Anyakoha and answer question 13 and 14 on page 121.

WEEK 4:

DATE: _____

TOPIC: EQUATION OF UNIFORMLY ACCELERATED MOTION

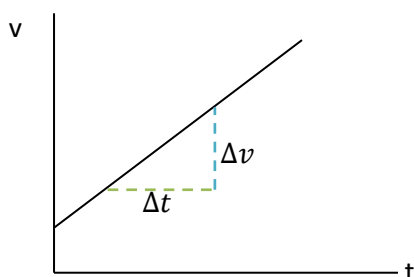
CONTENT

1. Revision on velocity-time graphs.
2. Derivations of equation of uniformly accelerated motion
3. Application of equation of uniform motion
4. Equation of motion under gravity

PERIOD ONE:

Velocity – time v-t graph.

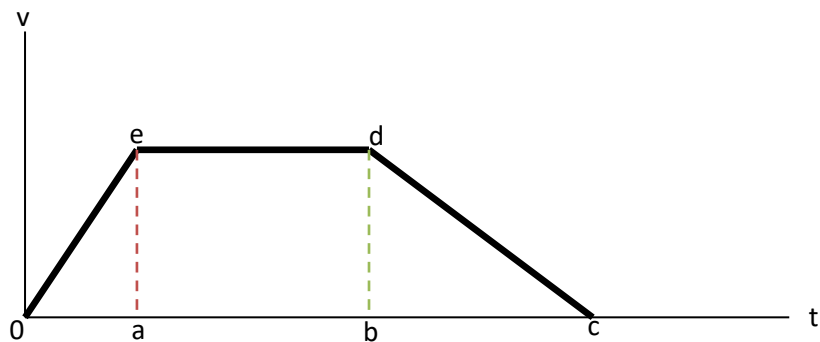
1. Gradient of a v-t graph = acceleration



$$\text{Acceleration} = \text{gradient} = \frac{\text{change in velocity}}{\text{change in time}}$$

$$\text{Acceleration } a = \frac{\Delta v}{\Delta t}$$

2. Area under a v-t graph = distance.



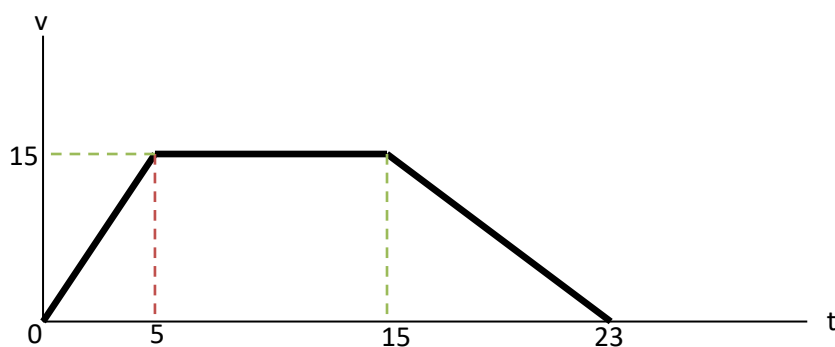
- i. Total distance covered during the motion = area of trapezium 0edc
- ii. Distance covered during acceleration = area of triangle 0ea
- iii. Distance covered during constant velocity = area of rectangle aedb
- iv. Distance covered during deceleration = area of triangle bdc
- v. Acceleration = slope of line 0e, $a = \frac{ae}{0a}$
- vi. Deceleration = slope of dc, $-a = \frac{bd}{bc}$

Example 1

1. A car starts from rest and accelerates uniformly to 15ms^{-1} in 5 s. it then continues at this velocity for the next 10s before decelerating back to rest in another 8 s.

Use the information to answer the following questions

- i. Sketch the velocity time graph of the motion of the car
- ii. Calculate the acceleration of the car
- iii. Calculate the deceleration of the car
- iv. What is the total distance travelled by the car
- v. Estimate the average speed of the car.



$$\text{ii. Acceleration } a = \frac{ae}{oa}$$

$$a = \frac{15}{5} = 3\text{ms}^{-2}$$

$$\text{iii. deceleration } -a = -a = \frac{bd}{bc}$$

$$-a = \frac{15}{23 - 15} = \frac{15}{8}$$

$$-a = -1.875\text{ms}^{-2}$$

iv. total distance = area under the graph
= area of trapezium

$$S = \frac{1}{2}(a + b)h$$

$$\frac{1}{2}\{(15 - 5) + (23 - 0)\}15$$

$$= \frac{1}{2}(10 + 23)15$$

$$= \frac{33 \times 15}{2} = \frac{495}{2}$$

$$s = 247.5\text{m}$$

$$\text{v. average speed } v = \frac{\text{total distance}}{\text{total time}}$$

$$v = \frac{247.5}{23} = 10.8\text{ms}^{-1}$$

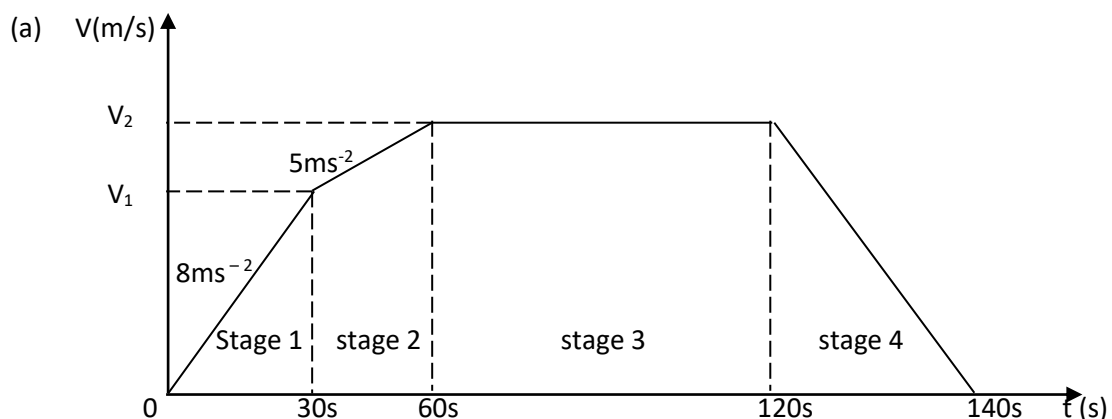
Example 2

A body at rest is given an initial uniform acceleration of 8.0ms^{-2} for 30s after which the acceleration is reduced to 5.0ms^{-2} for 30s. The body maintains the speed attained for 60s after which it is brought to rest in 20s.

(a) Draw the velocity-time graph of the motion using the information given above.

Using the graph, calculate (b) maximum speed attained during the motion. (c) average retardation as the body is brought to rest. (d) total distance travelled during the first 60s (e) average speed during the same intervals as in (c)

Solution.



(b) there are two stages of acceleration

Stage 1. Acceleration = gradient $a = 8 \text{ ms}^{-2}$

$$a = \frac{V_1 - 0}{30 - 0}$$
$$8 = \frac{V_1}{30}$$

Cross multiplying $V_1 = 8 \times 30 = 240 \text{ ms}^{-1}$

Stage 2. $a = 5 \text{ ms}^{-2}$

$$a = \frac{V_2 - V_1}{60 - 30}$$
$$5 = \frac{V_2 - V_1}{30}$$

Cross multiplying

$$V_2 - V_1 = 150$$

But $V_1 = 240$

$$V_2 - 240 = 150$$
$$V_2 = 150 + 240 = 390 \text{ ms}^{-1}$$

the maximum velocity is 390 ms^{-1}

(c)

Average retardation is equal to gradient

$$-a = \frac{V_2 - 0}{140 - 120} \quad \text{but } V_2 = 390 \text{ ms}^{-1}$$
$$-a = \frac{390 - 0}{20}$$
$$-a = 19.5 \text{ ms}^{-2}$$

Average retardation = -19.5 ms^{-2}

(d) distance is in the first 60 sec = area of triangle + area of the next trapezium

$$S = \frac{1}{2}(\text{time})V_1 + \frac{1}{2}(V_1 + V_2)\text{time}$$

$$S = \frac{1}{2}(30)(240) + \frac{1}{2}(240 + 390)30$$

$$S = 3600 + 9450 = 13050 \text{ m}$$

(d) average speed $V = \frac{\text{total distance}}{\text{total time}}$

$$V = \frac{13050}{60} = 217.5 \text{ ms}^{-1}$$

RELATIVE MOTION

This is the motion of a body with respect to another. All motion is relative. The motion of a car on the road is with respect to the earth or any other frame of reference in which the motion of the car is being observed.

Resultant velocity of relative motions

- Consider two cars X and Y travelling in the same direction and at the same speed, a commuter in X will observe that Y is stationary (not moving)

$$V_x = V_y$$

$$\text{Relative velocity} \quad V_x - V_y = 0$$

- If car X is to be travelling at a speed V_x which is greater than the speed of V_y , a commuter in car Y will observe the speed of car X to be

$$V_x - V_y = \text{relative velocity of car X with respect to Y}$$

A commuter in X will observe the relative velocity of Y to be

$$V_y - V_x$$

This value will be negative. This means that to an observer in X, the car Y will appear to be going backward (going the opposite direction with a speed of $|V_y - V_x|$)

- But if car X and Y were to be travelling in opposite directions, the relative velocity of X with respect to Y will be

$$V_x - V_y = \text{relative velocity of X with respect to Y}$$

$$V_y - V_x = \text{relative velocity of Y with respect to X}$$

N.B. note that the relative velocity of X with respect to Y, V_{xy} is equal in magnitude but opposite in direction to the relative velocity of Y with respect to X, V_{yx} .

$$V_{xy} = -V_{yx}$$

EXAMPLES

1. Two racing cars A and B travelling in the same direction at 300 m/s and 340 m/s respectively. What is the relative velocity of A with respect to B?

Solution:

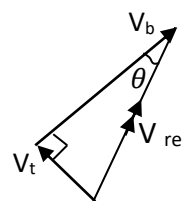
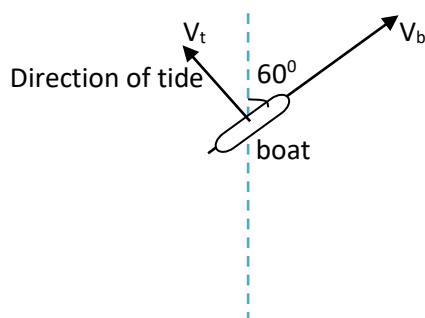
$$V_a = 300 \text{ km/s}$$

$$V_b = 340 \text{ km/s}$$

$$\text{Relative velocity of a with respect to B, } V_{ab} = V_a - V_b = 300 - 340 = -40 \text{ m/s}$$

(note that this is negative. A appears to be travelling in the opposite direction to B)

2. A boat whose speed is 8 km/h sets course on a bearing 060° . If the tide is running at a speed of 3 km/h from a bearing of 330° , find;
 - i. The actual speed of the boat (i.e., relative speed of the boat)
 - ii. The direction of travel



N.B the angle 90° in the triangle is obtained by geometry

To obtain the relative velocity (actual velocity), draw the component velocity such that the head of one point to the end of the other. Draw the relative velocity to begin from end of the first to the head of the last.

Using Pythagoras theorem

$$V_{rel}^2 = 8^2 + 3^2$$

$$V_{rel}^2 = 64 + 9 = 73$$

$$V_{rel} = \sqrt{73} = 8.54 \text{ kmh}^{-1}$$

Let θ be the angle between the relative velocity and the direction of the boat.

$$\tan \theta = \frac{V_t}{V_b} = \frac{3}{8}$$

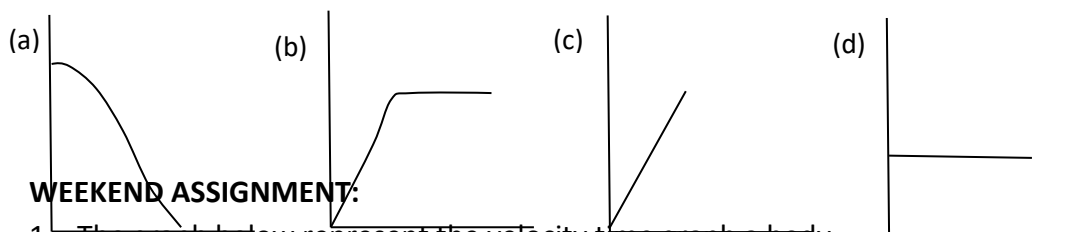
$$\theta = \tan^{-1}[0.375] = 20.6^\circ$$

EVALUATION

1. A train runs at a constant speed of 20m/s for 300s. and then accelerate uniformly to a speed of 30m/s over a period of 20s. this speed is maintained for 300s before the train is brought to rest with uniform deceleration is 30s. draw the velocity – time graph to represent the journey describe above. From the graph find,
 - i. The acceleration while the speed changes from 20m/s to 30m/s.
 - ii. The total distance travelled in the time described
 - iii. The average speed over the time described. (J.M.B)
2. A car travels at a uniform velocity of 20m/s for 4s. if the brakes were applied to bring the car to rest in the next 8 s. draw the velocity time graph for the motion. How far does the car travel after the brakes were applied?

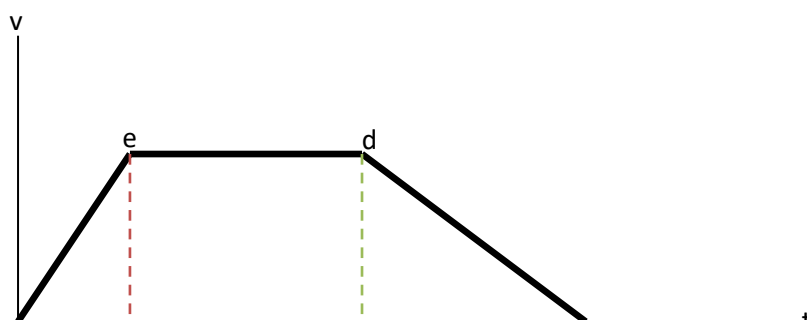
GENERAL EVALUATION

1. The planetary motions are examples of ----- motion. (a) rectilinear (a) rotation (c) vibratory (d) spin
2. The rate of change of velocity is called ---- (a) speed (b) displacement (c) uniform velocity (d) acceleration
3. Which of these is also referred to as negative acceleration? (a) instantaneous acceleration (b) uniform acceleration (c) retardation (d) non-uniform acceleration
4. The gradient of a distance –time graph gives ---- (a) velocity (b) acceleration (c) speed (d) displacement
5. ---- represent the area under a velocity – time graph. (a) distance (b) speed (c) acceleration (d) none of the above
6. Which of these graphs represent the velocity-time graph of the motion of a spherical metal ball falling through a fluid until it attain it terminal velocity?



WEEKEND ASSIGNMENT:

1. The graph below represent the velocity time graph a body



Sketch the corresponding displacement –time graph for this motion.

READING ASSIGNMENT

Students should read page 121-125 of the New School Physics by MA Anyakoha and answer question 21 and 25 on page 131

DERIVATIONS OF EQUATION OF UNIFORMLY ACCELERATED MOTION

Sub-Topic 3: ANALYSIS OF RECTILINEAR MOTION.

Supposing a body moving at an initial velocity ' u ' later attains a final velocity ' v ' in time t , Its acceleration is given as:

$$a = \frac{\text{change in velocity}}{\text{time taken}}$$

Change in velocity = $v - u$

$$\therefore a = \frac{v - u}{t} \text{ --- (1)}$$

By making v subject of the formula,

$$\text{We have } v = u + at \text{ --- (2)}$$

Recall that $\text{speed} = \frac{\text{distance}}{\text{time}}$

$$\therefore v = \frac{s}{t}$$

$$\therefore s = vt$$

Since the body above experienced two velocities, u & v , thus, the average velocity is $\frac{v+u}{2}$

$$\text{Hence, } s = \frac{(v+u)t}{2} \text{ --- (3)}$$

Putting (2) into (3), we have

$$s = \frac{(u + at + u)t}{2} = \frac{2ut + at^2}{2} = \frac{2ut}{2} + \frac{at^2}{2}$$

$$\text{Hence, } s = ut + \frac{1}{2}(at^2) \text{ --- (4)}$$

From (1),

$$t = \frac{v - u}{a} \text{ --- (5)}$$

Putting (5) into (3), we have

$$s = \frac{v+u}{2} \times \frac{v-u}{a} = \frac{(v+u)(v-u)}{2a}$$

But $(v+u)(v-u)$ is a difference of two squares, implying that $(v+u)(v-u) = v^2 - u^2$

Hence, $s = \frac{v^2 - u^2}{2a}$

Making v^2 subject of the formula, we have

$$v^2 = u^2 + 2as \quad \text{--- (6)}$$

Thus, in summary, the equations of motion include:

$$v = u + at$$

$$s = \left(\frac{v+u}{2}\right)t =$$

$$s = ut + \frac{1}{2}at^2$$

$$s = vt - \frac{1}{2}at^2$$

$$v^2 = u^2 + 2as$$

Under gravity, for a body descending, g is $+ve$. Therefore, the above equations become:

$$v = u + gt$$

$$s = ut + \frac{1}{2}gt^2$$

$$v^2 = u^2 + 2gs$$

Under gravity, for a body ascending, g is $-ve$. Therefore, the above equations become:

$$v = u - gt$$

$$s = ut - \frac{1}{2}gt^2$$

$$v^2 = u^2 - 2gs$$

EVALUATION

1. Define the following terms (i) average speed (ii) average velocity (iii) uniform acceleration (iv) constant velocity
2. State the value of the acceleration of a body moving with uniform velocity.

PERIOD TWO

Derivation of equation of uniform motion

Recall that,

Equation (i) --- $V = U + at$

Equation (ii) --- $S = \frac{(V+U)t}{2}$

Substituting equation (i) into equation (ii)

$$S = \frac{[(U + at) + U]t}{2s}$$

$$S = \frac{1}{2}[U + at + U]t$$

$$S = \frac{1}{2}[2U + at]t$$

$$S = \frac{1}{2}[2Ut + at^2]$$

$$S = Ut + \frac{1}{2}at^2 \dots \dots \dots (iii)$$

Again from equation (i),

$$V = U + at$$

$$V - U = at$$

Dividing both sides by a,

$$\frac{V - U}{a} = t \dots \dots \dots (3)$$

Substituting equation (3) into equation (ii)

$$(ii) \dots S = \frac{(V+U)}{2} t$$

becomes

$$S = \frac{(V + U)}{2} \times \frac{(V - U)}{a}$$

$$S = \frac{(V + U)(V - U)}{2a}$$

Expanding the bracket in the numerator,

$$S = \frac{V^2 - U^2}{2a}$$

Cross multiplying,

$$V^2 - U^2 = 2aS$$

$$V^2 = U^2 + 2aS \dots \dots \dots (iv)$$

$$V = U + at \dots \dots \dots (i)$$

$$S = \frac{(V + U)t}{2} \dots \dots \dots (ii)$$

$$S = Ut + \frac{1}{2}at^2 \dots \dots \dots (iii)$$

$$V^2 = U^2 + 2aS \dots \dots \dots (iv)$$

$$S = vt - \frac{1}{2}at^2 \dots \dots \dots (v)$$

Summarily, the equations of uniformly accelerating bodies are:

.N.B. **note that these equations can only be applied to solve problems on *bodies moving with constant/uniform acceleration***. Problems on bodies moving with non-uniform acceleration can be solved using differential calculus.

EVALUATION

1. State the equations of uniformly accelerating bodies.
2. Derive equation (iv) of uniformly accelerating motion.

PERIOD THREE

Application of the equations of uniform accelerating bodies.

1. A train starts from rest and accelerate until it attains a velocity of 8m/s is 10 s. calculate the acceleration of the train.

Solution:

For a body at rest velocity is zero.

Initial velocity $U=0$

Final velocity $V= 8\text{m/s}$

Time $t=10\text{ s}$

Acceleration $a= ?$

{ you use any of the four equations that has U,V, t, a has identified from the question }

$$V = U + at$$

$$8 = 0 + a \times 10$$

$$8 = 10a$$

Dividing both side by 10

$$a = 0.8\text{m/s}^2$$

2. A horse rider moving with constant acceleration covers the distance between two point 70.0m apart in 7.0 s. if his speed as he passes the second point is 15.0 m/s. what is its speed at the first point?

Distance $S = 70.0\text{m}$

Time $t = 7.0\text{s}$

Initial speed $U = ?$

Final speed $V = 15.0\text{m/s}$

{ the equation containing S, t, U, and V is $S = \frac{(V+U)t}{2}$ }

$$S = \frac{(V + U)t}{2}$$

$$70 = \frac{(15+U)7}{2}$$

Cross multiplying

$$(15+U)7 = 140$$

Dividing both sides by 7

$$15 + U = 20$$

$$U = 20 - 15$$

$$U = 5\text{m/s}$$

3. A body starts with an initial velocity of 26m/s and moves down it with uniform acceleration of 7m/s^2 for 25 s. find the total distance moved in metres

Solution: Initial velocity $U = 26\text{m/s}$

Acceleration $a = 7\text{m/s}^2$

Time $t = 25\text{ s}$

Distance $S = ?$

{the equation containing U, a, t and s is $S = Ut + \frac{1}{2}at^2$ }

$$S = Ut + \frac{1}{2}at^2$$

$$S = 26 \times 25 + \frac{1}{2} \times 7 \times 25^2$$

$$S = 650 + 3.5 \times 625$$

$$S = 650 + 2187.5$$

$$S = 2837.5 \text{ m}$$

Motion of bodies under gravity

Neglecting air resistance, motion of bodies moving under gravity (either vertical upward or downward) is an example of uniformly accelerating motion.

1. A body thrown vertically upward in the earth gravitational field.

When a body is thrown vertically upward from the earth surface, it retards uniformly (with acceleration of $a = -g$) until it attains its maximum height where its final velocity is zero. ($V = 0$)

If U is the initial velocity with which the body was projected vertically upward and $H=S$ is the maximum height where its velocity is zero (i.e., temporarily at rest before coming down)

g – acceleration due to gravity

$$V^2 = U^2 + 2aS$$

$V=0$ $s = H$ $a = -g$ is negative (retardation) where g is the acceleration due to gravity

$$0 = U^2 + 2(-g)H$$

$$0 = U^2 - 2gH$$

$$2gH = U^2$$

$$H = \frac{U^2}{2g} \quad H \text{ is the maximum height}$$

Again, using $V = U + at$

$$V = 0 \quad a = -g$$

$$0 = U + (-g)t$$

$$0 = U - gt$$

$$gt = U$$

$$t = \frac{U}{g}$$

t is the time to reach the maximum height.

If the body is thrown vertically upward and allowed to return to the point of projection, the total time of flight is given as

$$t = \frac{2U}{g}$$

2. Motion of a bodies falling freely under gravity

The body was initially at rest, hence the initial velocity is zero. As it falls, its velocity increases i.e. it accelerates, $a = g$

$$\text{Using,} \quad V^2 = U^2 + 2aS$$

$$V^2 = 0 + 2gH$$

$$V = \sqrt{2gH}$$

This is the velocity of the body just as it is about to reach the ground

Again using

$$S = Ut + \frac{1}{2}at^2$$

$$H = 0 \times t + \frac{1}{2}gt^2$$

$$H = \frac{1}{2}gt^2$$

$$t = \sqrt{\frac{2H}{g}}$$

this is the time to reach the ground

CLASSWORK; A projectile is fired vertically upward and it reach a height of 78.4 m. find the velocity of projection and the time it takes to reach the highest point.(take $g = 10\text{m/s}^2$)

Solution: initial velocity U = velocity of projection

$$U = ?$$

$$\text{Maximum height } S = H = 78.4 \text{ m}$$

$$\text{Acceleration } a = -g = -10\text{m/s}^2$$

Final velocity $V = 0 \text{ m/s}$ (body is temporarily at rest at the maximum height)

$$\{U, S, a, V\}$$

$$V^2 = U^2 + 2aS$$

$$V^2 = U^2 + 2(-g)H$$

$$0 = U^2 - 2 \times 10 \times 78.4$$

$$U^2 = 1568$$

$$U = \sqrt{1568}$$

$$U = 39.6 \text{ m/s}$$

$$V = U - gt$$

$$0 = 39.6 - 10t$$

$$t = 3.96\text{s}$$

CLASSWORK; A body falls from a height of 80m. What is it velocity just before hitting the ground

Solution; height $H = S = 80\text{m}$

Initial velocity $U = 0$ (body is taken be initially at rest)

Acceleration $a = g = 10 \text{ m/s}^2$ (this is positive because the body is coming down)

Final velocity $V = ?$

$$\{S, U, a, V\}$$

$$V^2 = U^2 + 2aS$$

$$V^2 = 0^2 + 2 \times 10 \times 80$$

$$V^2 = 1600 = 40\text{ms}^{-1}$$

A stone is dropped from a height of 196 m. neglecting air resistances; calculate the time to reach the ground.

Solution

$$H=S = 196 \text{ m}$$

$a = g = 10 \text{ m/s}^2$. (g is positive because the body is moving downward)

$U=0\text{m/s}$ (body is taken to be initially at rest)

$$t = ?$$

$$S = Ut + \frac{1}{2}at^2$$

$$196 = 0 \times t + \frac{1}{2} \times 10 \text{ t}^2.$$

$$196 = 5 t^2.$$

$$t^2 = \frac{196}{5}$$

$$t^2 = 39.2$$

$$t = \sqrt{39.2}$$

$$t = 6.3 \text{ s}$$

EVALUATION:

1. A stone was thrown vertically upward with an initial speed U . If g is the acceleration of free fall, show that the time taken for the ball to return to its point of projection is $\frac{2U}{g}$
2. A ball is thrown vertically upward with a velocity of 19.6m/s. what distance does it travels before coming to rest momentarily at the maximum height?
3. With what velocity must a ball be projected vertically upward for it return to it point of projection in 5s?
4. A vehicle which starts from rest is accelerated uniformly at the rate of 5m/s^2 for 5 s. It attains a speed which is maintained for 60 s. the vehicle is then brought to rest by a uniform retardation after another 3 s. determine the total distance covered.

GENERAL EVALUATION

1. Two cars A and B move parallel to each other but in opposite direction. If the velocity of A is 10m/s and that of B is 15m/s. what is the relative velocity of B with respect to A?
2. A car travelling with a uniform acceleration of 3m/s^2 starts from rest. What time will it attain a velocity of 15m/s?

WEEKEND ASSIGNMENT

Derive an equation of uniformly acceleration motion that involves only S, V, t and a

READING ASSIGNMENT

Students should read pages 125, 130-132 of New School Physics. By MA Anyakoha and answer question 26 on page 131

TOPIC; PROJECTILE

CONTENT

1. Concept of projectiles
2. Example of projectile motion
3. Components of projectile.
4. Horizontal projection

PERIOD ONE

CONCEPT OF PROJECTILE

Projectile is the motion of a body which travels freely in space under the influence of gravity and air resistance. When a ball is kicked into the air, it will travel through space in a plane. The motion in a plane is a combination of upward and horizontal motion.

The path through which a projectile travels is called ***trajectory***.

Example of projectile

In sport,

- Thrown discus
- Thrown javelin

In warfare

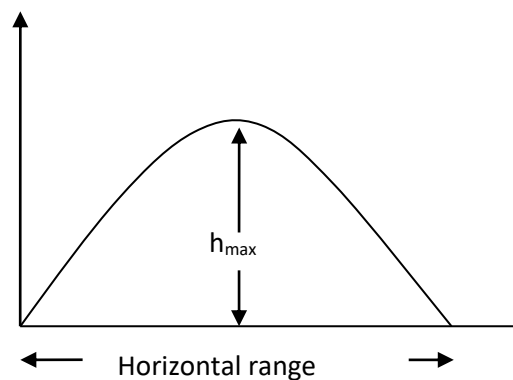
- Fired stone from a catapult
- Shot arrow from a bow
- Launched missiles

Miscellaneous

- Thrown stones

Components of projectile

When a body is projected at an angle θ to the horizontal, the trajectory is a parabola.

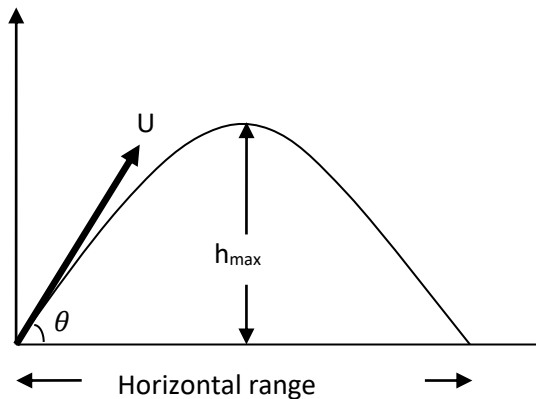


The motion of this projectile can be split into two:

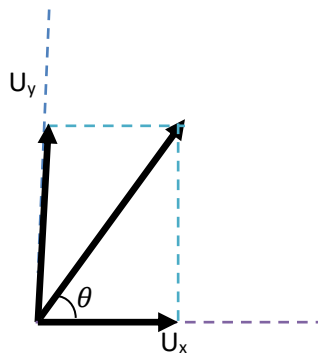
- I. The horizontal motion

In the horizontal motion, the body moves with constant velocity. Therefore the horizontal acceleration is zero. This also implies that the initial and the final horizontal velocity are equal.

$$a_x = 0 \quad V_x = U_x$$



U is the initial velocity with which the body was projected. Resolving U into its vertical and horizontal components, we have:



1. $U_x = U \cos \theta$
2. $U_y = U \sin \theta$

$$\therefore U_x = V_x = U \cos \theta$$

the horizontal Range R , is the horizontal distance covered by the projectile.

Since the acceleration along the horizontal is zero,

Horizontal Range R , $R = U_x t$

$$R = (U \cos \theta) t$$

$$R = Ut \cos \theta \dots \dots \dots (i)$$

2. Vertical motion

The vertical motion is an example of a uniformly accelerated motion. The equations of uniform motion are still valid for it.

During the upward motion,

Vertical acceleration $a_y = -g$ (where g is the acceleration due to gravity)

Initial vertical velocity $U_y = U \sin \theta$

At the maximum height, the body is temporarily at rest. Therefore $V_y = 0$

Substituting these into

$$V = U + at$$

$$V_y = U_y + at$$

$$0 = U \sin \theta + (-g)t$$

$$0 = U \sin \theta - gt$$

$$gt = U \sin \theta$$

$$t = \frac{U \sin \theta}{g} \dots \dots \dots (ii)$$

t is the time to reach the maximum height.

The total time of flight is twice the time to reach the maximum height

Total time of flight T, $T = \frac{2U \sin \theta}{g} \dots \dots \dots (iii)$

Using the equation, $V^2 = U^2 + 2aS$

$$V_y^2 = U_y^2 + 2(-g)S_y$$

$$0 = (U \sin \theta)^2 - 2gH_{max}$$

$$2gH_{max} = U^2 \sin^2 \theta$$

$$H_{max} = \frac{U^2 \sin^2 \theta}{2g} \dots \dots \dots (iv)$$

Summarily

$$R = Ut \cos \theta \dots \dots \dots (i)$$

$$t = \frac{U \sin \theta}{g} \dots \dots \dots (ii)$$

$$T = \frac{2U \sin \theta}{g} \dots \dots \dots (iii)$$

$$H_{max} = \frac{U^2 \sin^2 \theta}{2g} \dots \dots \dots (iv)$$

The horizontal range R can also be expressed as

$$R = \frac{U^2 \sin 2\theta}{g} \dots \dots \dots (v)$$

For range to be maximum, 2θ must be equal to 90° .

Therefore maximum range occur when $\theta = 45^\circ$

$$R_{max} = \frac{U^2}{g} \dots \dots \dots (vi)$$

	Vertical component	Horizontal component
Initial velocity	$U \sin \theta$	$U \cos \theta$
Velocity at the any point p	V_y	V_x
Velocity at the max height	0	$U \cos \theta$
Displacement at any point p	S_y	$S_x = Ut \cos \theta$
acceleration	$-g$	0

CLASSWORK:

1. A projectile is fired from the ground level with a velocity of 500m/s at 30° to the horizontal. Determine;
 1. It horizontal range
 2. The greatest height attained.

SOLUTION:

$$\theta = 30^\circ$$

$$U = 500 \text{ms}^{-1}$$

$$R = ?$$

$$R = \frac{U^2 \sin 2\theta}{g}$$

$$R = \frac{500^2 \sin(2 \times 30)}{10}$$

$$R = \frac{250000 \sin 60}{10}$$

$$R = 25000 \times 0.866$$

$$R = 2165 \text{m}$$

$$(i) \quad H_{max} = \frac{U^2 \sin^2 \theta}{2g}$$

$$H_{max} = \frac{500^2 \sin^2 30}{2 \times 10}$$

$$H_{max} = \frac{250000 \times (\sin 30)}{20}$$

$$H_{max} = 12500 \times 0.5$$

$$H_{max} = 6250 \text{m}$$

2. A bullet is fired at an angle of 45° to the horizontal with a velocity of 450m/s . calculate (i) time to reach the maximum height (ii) the maximum height reached and the horizontal distance from the point of projection at this instant.

SOLUTION

$$\begin{aligned} (i) \quad \theta &= 45^\circ \\ U &= 450\text{ms}^{-1} \\ g &= 10\text{ms}^{-2} \quad t = ? \\ t &= \frac{U \sin \theta}{g} \end{aligned}$$

$$t = \frac{450 \sin 45}{10}$$

$$t = 45 \times 0.7071$$

$$t = 31.8 \text{ s}$$

(Further examples should be solved as classwork)

EVALUATION

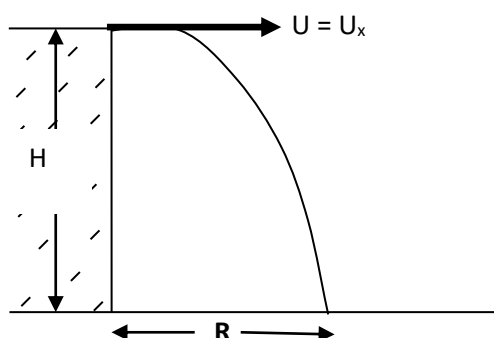
A particle is projected from the ground level with a velocity of 40m/s at an angle of $\tan \theta = \frac{3}{4}$. Calculate the

- I. Time of flight
- II. Range
- III. Time taken to reach the greatest height
- IV. Greatest height

PERIOD TWO

BODIES PROJECTED HORIZONTALLY AT A HEIGHT ABOVE THE GROUND

The motion of such projectile can also be split into two: the horizontal and the vertical motion.



1. Horizontal motion

In the horizontal motion, the body moves with constant velocity. Therefore, the horizontal acceleration is zero. This also implies that the initial and the final horizontal velocity are equal.

$$U_x = V_x = U$$

$$a_x = 0.$$

If t is the time to reach the ground, then $R = Ut \dots \dots \dots (vii)$

2. Vertical motion

The vertical motion is an example of a uniformly accelerated motion. The equations of uniform motion are still valid for it.

During the upward motion,

Vertical acceleration $a_y = -g$ (where g is the acceleration due to gravity)

The body was given an initial horizontal velocity. Since no vector has a perpendicular component, $U_y = 0$

Height $H = S_y$

Substituting these into $S = Ut + \frac{1}{2}at^2$.

$$H = 0 \times t + \frac{1}{2}gt^2$$

$$H = \frac{1}{2}gt^2$$

making time the subject of the formulae

$$t = \sqrt{\frac{2H}{g}} \dots \dots \dots (viii)$$

This is the time taken to reach the ground or foot of the height

This time t is the same for both the vertical and horizontal bodies dropped from a given height.

CLASSWORK

1. A ball is projected horizontally from the top of building with a velocity of 10m/s. the height of the building is 45m. determine;
 - i. Time taken by the ball to reach the ground
 - ii. Distance of the ball from the building after hitting the ground
 - iii. The direction of the ball to the horizontal just before it hit the ground.

EVALUATION

1. For a particular value of U , at what value of θ to the horizontal should a ball be projected in order to have a maximum range? (a) 15° (b) 30° (c) 45° (d) 60°
2. Which of these is not true about the horizontal motion of a projectile? (a) constant acceleration (b) uniform velocity (c) it is not affected by gravity (d) acceleration is zero
3. A coin is pushed from the edge of a laboratory bench with a horizontal velocity of 15m/s. If the height of the bench from the floor is 1.5m, calculate the distance from

the foot of the bench to the point of impact with the floor. ($g = 10\text{m/s}^2$) (a) 0.75m (b) 2.25m (c) 8.22 m (d) 15.00m. **(WASSCE 2011 Q13)**

WEEKEND ASSIGNMENT

1. Using any of the equation of uniform motion, such that the maximum height attained by a ball projected at an angle θ to the horizontal with a velocity U is

$$H_{max} = \frac{U^2 \sin^2 \theta}{2g}$$

READING ASSIGNMENT.

Students should answer the question on page 134 of New School Physics by MA Anyakoha

WEEK SIX

DATE

TOPIC: EQUILIBRIUM OF FORCES

CONTENT

1. Resultant of force and equilibrant force
2. Equilibrium and types of equilibrium
3. Moment of force and couple
4. Conditions for equilibrium for system of parallel forces
5. Centre of gravity (types of equilibrium) and centre of mass

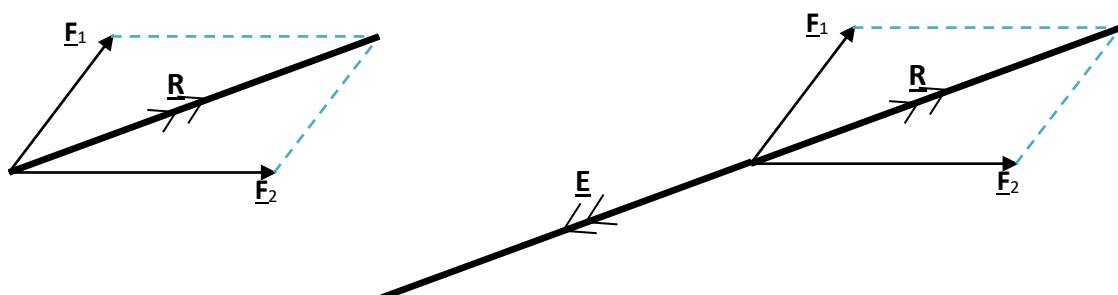
PERIOD ONE;

Resultant force

This can be defining a single force which can produce the same effect as the combined force on a system. The addition of two or more force produces the resultant force. The resultant of any system of force can be obtained through any of the process described earlier.

Equilibrant force

This is that force which when added to a system of vectors will make the resultant of the system zero. Equilibrant has the same magnitude as the resultant force but it always acts in a direction opposite to that of the resultant.



$$\text{resultant}(\mathbf{R}) = -\text{equilibrant}(\mathbf{E})$$

For a system in equilibrium, the resultant force is ZERO.

Types of equilibrium

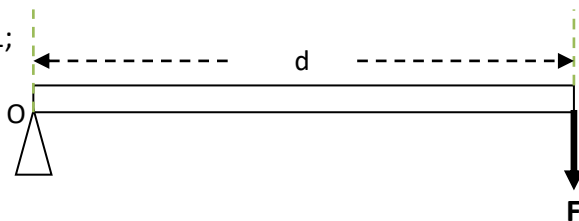
1. Stable equilibrium; a body is in stable equilibrium if its velocity and its resultant force is zero. ($v = 0$ and $R = 0$)
2. Dynamic equilibrium: a body is said to be in dynamic equilibrium if its velocity is constant or it is rotating with a constant angular velocity. For bodies in dynamic equilibrium, velocity is not zero but the resultant force on it is zero
(i.e., $V \neq 0$ but $\text{Resultant} = 0$)
3. Translational equilibrium: a body is said to be in translational equilibrium if there is no net force acting on it though it is at rest or moving with constant velocity.

Thus a body is said to be in equilibrium if its resultant force is zero.

Moment of a force

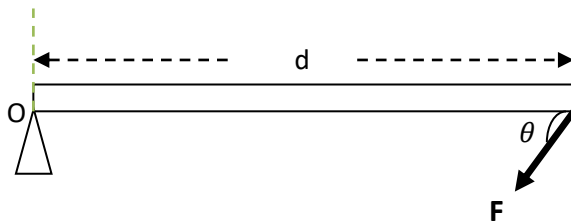
The turning effect of a force is its moment. **Moment of a force about a point can be defined as the product of the force and its perpendicular distance from the point.**

CASE 1;



Moment of the force F about the point O = $F \times d$

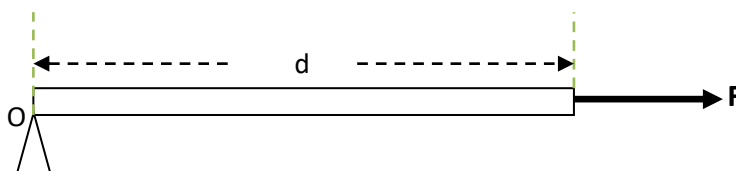
CASE 2.



The perpendicular component of F is $F \sin \theta$. So moment of F about O is

Moment = $F d \sin \theta$

CASE 3:



The force F will create a translational motion and not a turning effect. Therefore the moment of F in this case is **ZERO**.

N.B note that the moment of a force is maximum when the force is at right angle.

EVALUATION

1. The product of a force and its perpendicular distance from a point called ---- (a) resultant (b) equilibrant (c) moment of force (d) couple
2. If moment $M = Fd \sin \theta$, for what value of θ will the moment of the force F be zero? (a) 0° (b) 90° (c) 180° (d) 270° .
3. The angle between the resultant and the equilibrant of a system of force is ---- (a) 0° (b) 90° (c) 180° (d) 270° .
4. The type of equilibrium possessed by a body falling through a fluid after attaining its terminal velocity is ---- equilibrium. (a) stable (b) unstable (c) dynamic (d) neutral
5. The resultant force on a body in translational equilibrium is ---- (a) $= 0$ (b) < 0 (c) > 0 (d) ≈ 0

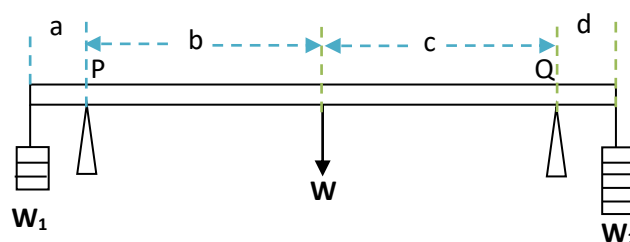
PERIOD TWO

Principle of moment

This states that **for a system in equilibrium, the algebraic sum of moments about any point is zero.**

It can also be stated thus, **for a system in equilibrium, the sum of the clockwise moment about a point is equal to the sum of the anticlockwise moment about the same point.**

Consider the system below,



Three downward forces, W_1 , W , W_2 .

Reaction at P and Q, constitute the two upward forces acting on the body.

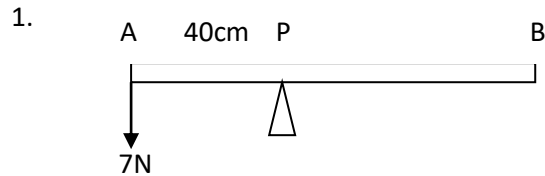
Taking moment about P,

clockwise moment		Anticlockwise moment	
i.	$W \times b$	i.	$R_Q \times (b + c)$
ii.	$W_2 \times (b + c + d)$	ii.	$W_1 \times a$

Don't forget moment of a force is **Force** x perpendicular distance.

Classwork: following the example above, take moment about the point Q and write out the clockwise and the anticlockwise moments.

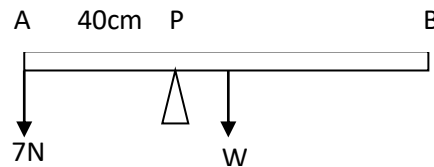
Examples;



In the diagram above, AB represent a uniform rod of length 1.50m which is in equilibrium on a pivot at p. if AP = 40cm, calculate the mass of the rod. ($g = 10\text{ms}^{-2}$)

Solution:

Since the rod is uniform, its weight acts at the centre of the rod



the rod is 1.5m long, its centre is $\frac{1.5}{2} = 0.75\text{m}$ from one of its ends

taking moment about the pivot

clockwise moment = $f \times d = 7\text{N} \times 0.4\text{m} = 2.8\text{ Nm}$

Anticlockwise moment = $f \times d = W \times (0.75 - 0.4) = 0.35W$

At equilibrium, **clockwise moment = anticlockwise moment**

$$0.35W = 2.8$$

$$W = \frac{2.8}{0.35} = 8.0\text{N}$$

The weight of the rod is 8.0N

But weight $W = mg$

$$8 = m \times 10$$

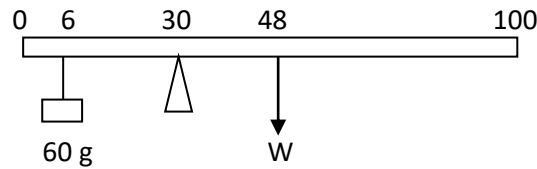
$$m = \frac{8}{10} = 0.8\text{ Kg}$$

Mass of the rod is 0.8 kg

2. A metre rule is found to balance at 48 cm marked. When a body of 60 g is suspended at 6 cm, the balance point is found to be 30 cm.

- i. Calculate the mass of the rule.
- ii. What is the new balance point if the 60 g is moved to 13 cm mark.

Solution



W is mass of metre rule

N.B

- a metre rule is 100 cm long
- for uniform metre rule the weight (position of c.g) is 50 cm mark
- for non-uniform metre rule, c.g is at the balance point when no load is on the rod

(i) taking moment about the pivot

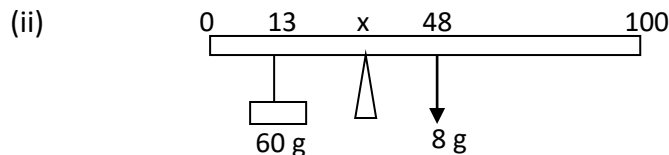
$$\text{clockwise moment} \quad f \times d = 60 \times (30 - 6) = 60 \times 24 = 144$$

at equilibrium, clockwise moment = anticlockwise moment

$$18W = 144$$

$$W = \frac{144}{18} = 8 \text{ g}$$

The mass of the metre rule is 8 g.



The 60 g is now at 13 cm mark, the new balance point is x the cg still remains 48 cm.

Taking moment about the pivot

$$\text{Clockwise moment} \quad 60 \times (x - 13)$$

$$\text{Anticlockwise moment} \quad 8 \times (48 - x)$$

At equilibrium, clockwise moment = anticlockwise moment

$$60(x - 13) = 8(48 - x)$$

$$60x - 780 = 384 - 8x$$

$$60x + 8x = 384 + 780$$

$$68x = 1164$$

$$x = 17.1\text{cm}$$

The new balance point is 17.1 cm mark

EXPERIMENT 3:

- An experiment to verify the principle of moment using the metre rule, standard mass and the spring balance.

Conditions for equilibrium for a system of parallel forces

1. Sum of forces in one direction must equal to the sum of forces in the opposite direction. Sum of upward forces must equal to the sum of downward forces.
2. Resultant force must be zero
3. The algebraic sum of moment about a point must be equal to zero.

However, for three non parallel co-planar forces to keep a body in equilibrium,

1. The line of action of the three forces must intersect at a point
2. The three forces can be represented in magnitude and direction by the adjacent sides of a triangle by taking the head of one to the tail of the other.

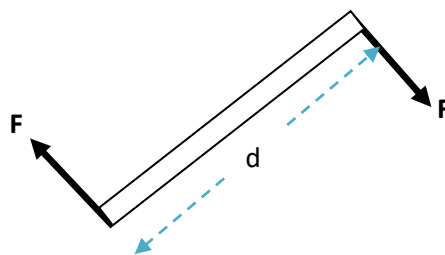
EVALUATION

1. State the principle of moment
2. State the condition of equilibrium for a system of parallel forces

PERIOD THREE.

Couple

A couple is a system of equal and opposite forces acting at a distance apart and whose lines of actions do not coincide. A couple will always create a turning effect about a point midway between the two forces. The forces of a couple create a torque. Couple cause an angular acceleration.



The moment of a couple = $F \times d$

The perpendicular distance between the two forces is called the **arm of the couple**. The moment of a couple about any point in a plane containing the two forces is the same. Moving the couple from one point to another in the plane does not change the value of the moment of the couple.

Application of couple

1. Turning of a tap head
2. Turning the stirring wheel of a car with two hands
3. Action of a corkscrew or
4. Use screwdriver to loosen a screw.
5. Action on a circular door knob

CLASSWORK: Two force of 10N each act at the opposite end of ruler 50cm long. Calculate the resultant force and the moment of the force.

SOLUTION:

(i) Assuming the two forces are parallel

$$\text{Resultant force} \quad R = 0 \text{ N}$$

(ii) The question above is an example of a couple

$$F = 10\text{N} \quad d = 50\text{cm} = 0.5\text{m}$$

$$\text{Moment of a couple} \quad M = F \times d$$

$$M = 10 \times 0.5 = 5 \text{ Nm}$$

Centre of gravity

This can be defined as a point on a body through which the line of action of the resultant weight of the body passes through. It is the point on an object where the resultant weight of the body is acting.

The position of the centre of gravity of an object can be determined through the following methods:

- i. Balancing method
- ii. Plumb line methods

EXPERIMENT 4-5:

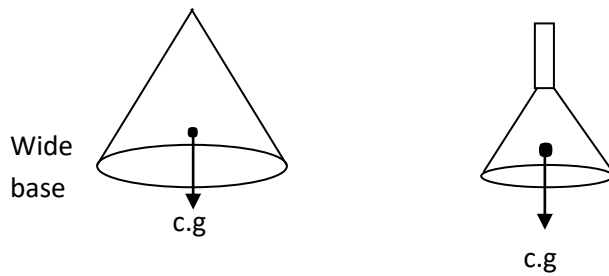
- an experiment to determine the centre of balance of metre rule using the balancing method.
- An experiment to determine the centre of gravity of a laminar irregular cardboard using the plumb line method

Uniform objects often have their centre of gravity at their midpoint / centre.

The position of the centre of gravity of an object determines the stability of the object.

TYPES OF EQUILIBRIUM / STABILITY

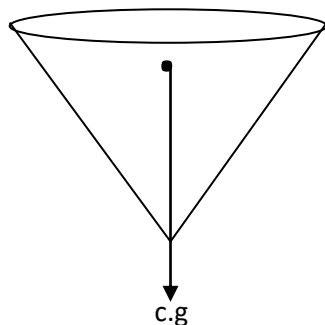
1. **Stable equilibrium:** bodies in stable equilibrium
 - i. Have centre of gravity close to their base (low c.g)
 - ii. Have wide base
 - iii. Returns to their original position after a slight tilt.



Examples of bodies in stable equilibrium; a cone sitting on its base, a funnel set upside down on a table

N.B when bodies in neutral equilibrium are slightly tilted, their potential energy increases but the line of action remains within the base.

2. **Unstable equilibrium:** bodies in unstable equilibrium
 - i. Have centre of gravity high above the base.
 - ii. Have narrow base
 - iii. Fall away from their original position when they are slightly tilted

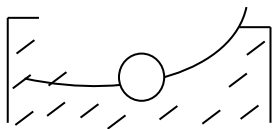


Example of bodies in unstable equilibrium is cone resting on its vertex.

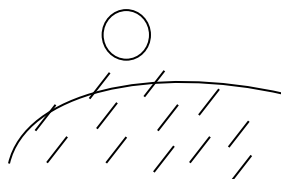
N.B when bodies in neutral equilibrium are slightly tilted, their potential energy decreases and the line of action falls outside the base.

3. **Neutral equilibrium:** For bodies in neutral equilibrium, the potential energy remains unchanged when they are slightly tilted.

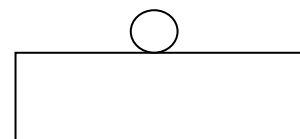
the diagram below typify the types of equilibrium.



Ball in stable equilibrium



Ball in unstable equilibrium



Ball in neutral equilibrium

(you can easily identify the type of equilibrium by considering how a body will fall off its equilibrium position when it is slightly tilted)

Centre of mass

This can be defined as the point on an object where the application of a force will produce acceleration and not a turning effect.

EVALUATION

1. Mention two example each of bodies in (i) stable equilibrium (ii) unstable equilibrium (iii) neutral equilibrium
2. Define a couple.

WEEK 7: MID-TERM BREAK

WEEK 8:

DATE

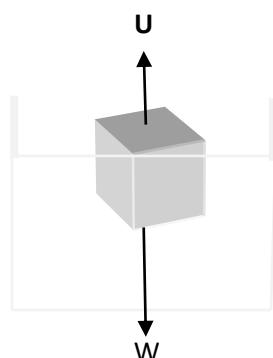
TOPIC: EQUILIBRIUM OF BODIES IN LIQUIDS

CONTENT:

- **Concept of Upthrust**
- **Archimedes Principle**
- **Density & Relative density**
- **Principle of flotation**

EQUILIBRIUM OF BODIES IN LIQUIDS

Boat, ship or a swimmer can float on water. This is as a result of certain forces acting on these bodies.



Consider a cube floating in water as shown below. For the cube to be in equilibrium $U = W$

The force U is called the upthrust.

Upthrust can be defined as an upward force experienced by an object in a fluid.

Upthrust can also be defined as the loss of weight experienced by an object partially or completely immersed in a fluid. For an object floating in a fluid,

$$\text{weight} = \text{upthrust}$$

$$W = U$$

For an object partly or wholly immersed in a fluid, (e.g. bucket of water inside the water in a well, weight lighter than)

$$U = \text{weight loss}$$

Consider a bucket of water of weight W in a well which is held by a string whose tension is T . When the bucket is above the water in the well, the tension in the string equals the weight of the bucket. ($W = T$)

When the bucket is inside the well, it experiences a weight loss which is equal to the difference ($W - T$)

$$\text{upthrust } U = W - T \dots \dots \dots (i)$$

Where W is the weight of the bucket in air and T is the bucket in the well/fluid.

EXPERIMENT 6

- To measure the upthrust experienced by a body immersed in water using the spring balance, eureka can and a beaker.

Archimedes' principle

This states that ***when a body is partly or completely immersed in a fluid, it experiences an upthrust which is equal to the weight of the fluid displaced.***

$$\text{upthrust } U = \text{weight of fluid displaced}$$

Weight = mass $\times$ acceleration due to gravity

Weight of fluid displaced $W = mg$

But density of the fluid, ρ , $\rho = \frac{m}{v}$

Where v – volume of fluid displaced.

$$m = \rho v$$

Weight of fluid displaced $W = mg = (\rho v)g$

$$\therefore \text{upthrust } U = \rho v g \dots \dots \dots (ii)$$

ρ – density of the fluid

v – volume of the fluid displaced

g - acceleration due to gravity

Density of a body

This can be defined as the ratio of the mass of body to its volume or mass per unit volume. In the laboratory, the density of a substance can simply be determined by measuring the mass of the substance using a triple balance and measuring the volume. With the mass and volume of the substance known, the density can be determined using:

$$\text{density } \rho = \frac{m}{v} \dots \dots \dots (iii)$$

Density is a scalar quantity and its S.I unit is kgm^{-3} . Another unit for density is gcm^{-3} .

$$1000 \text{ kgm}^{-3} = 1 \text{ gcm}^{-3}$$

Relative density

The relative density of a substance is the ratio of the density of the substance to the density of water. This has no unit. It can also be easily determined by estimating the density of the substance in kgm^{-3} and dividing it by 1000 kgm^{-3} (the density of water or in g/cm^3 and dividing by 1 gcm^{-3})

$$\text{relative density of a substance} = \frac{\text{density of the substance}}{\text{density of water}}$$

$$R.D = \frac{\rho_s}{\rho_w}$$

Relative density of a substance can also be defined as the ratio of the mass of the substance to the mass of equal volume of water.

$$R.D = \frac{\text{mass of substance}}{\text{mass of equal volume of water}}$$

The relative density of a liquid can be defined as the ratio of the upthrust experienced by an object in the liquid to the upthrust experienced by the object in water.

$$R.D \text{ of liquid} = \frac{\text{upthrust experienced by a body in liquid}}{\text{upthrust experienced by the same body in water}}$$

EXPERIMENTS 7-11

- Experiment to determine the relative density of a liquid using the relative density bottle
- Experiment to measure the density of regular solid
- Experiment to measure the density of irregular solid using the eureka can

- Experiment to measure the density of liquids using the measuring cylinder and triple balance
- Experiment to demonstrate weight loss by an object immersed in a fluid using the spring balance.

PRINCIPLE OF FLOATATION

The **law of floatation** states that for a body to float in a fluid, it must displace an amount of fluid equal to its own weight.

Weight of object = weight of fluid displaced.

$$\text{weight of object, } W_o = \rho v g \dots \dots \dots (iii)$$

Application of the principle of floatation

1. hydrometer
2. Submarine
3. Ship/boat
4. Hot air balloon
5. Floating iceberg

CLASSWORK

1. A body of mass 20g appears to have a mass of 13g in oil and 12g in water. What is the relative density of oil?

SOLUTION

$$\begin{aligned} \text{mass of object } m_o &= 20g \\ \text{mass of object in oil } m_1 &= 13g \\ \text{mass of object in water } m_w &= 12g \\ \text{upthrust in oil} &= m_o - m_1 = 20 - 13 = 7g \\ \text{upthrust in water} &= m_o - m_w = 20 - 12 = 8g \\ \text{relative density of oil} &= \frac{\text{upthrust in oil}}{\text{upthrust in water}} = \frac{7}{8} \end{aligned}$$

R.D of oil = $\frac{7}{8}$

2. A metal block of density 900 kg m^{-3} weighs 60N in air. find its weight when it is completely immersed in paraffin wax of density 800 kg m^{-3} ($g = 10 \text{ ms}^{-2}$)

Solution:

$$\text{Density of the object } \rho_o = \frac{m}{v} \rho_o = 900 \text{ kg m}^{-3}$$

Cross multiplying,

$$\text{Mass of object } m_o = \rho_o v \dots \dots \dots \text{ I}$$

$$\text{weight of object } W_o = m_o g \quad W_o = 60 \text{ N}$$

$$\text{Mass of object } m_o = \frac{W_o}{g} \dots \dots \dots \text{ II}$$

$$\text{Equating I and II } \rho_o v = \frac{W_o}{g}$$

$$900 \times v = \frac{60}{10}$$

$$9000 \times v = 60$$

$$v = \frac{60}{9000} = 0.007m^3$$

Recall, eqn ii

$$U = ?$$

$$\text{upthrust } U = \rho_l v g \quad \dots \dots \dots (ii)$$

$$\text{density of liquid } \rho_l = 800kgm^{-3}$$

$$U = 800 \times 0.007 \times 10 = 56N$$

But upthrust = weight loss

$$U = W_o - T$$

$$56 = 60 - T$$

$$T = 60 - 56$$

$$T = 4N$$

Weight of the block in the paraffin wax = 4N

further examples should be solved as classwork)

EVALUATION:

A piece of wood of mass 60 kg and density 600 kgm^{-3} float in water of density 100 kgm^{-3} . Calculate;

- (i) Volume of water displaced by the wood
- (ii) Fraction of the volume of the wood immersed in water

WEEKEND ASSIGNMENT

1. Differentiate between a resultant force and a equilibrant.
2. Mention two differences between centre of gravity and centre of mass.
3. A pencil of mass 5 g can be balanced horizontally on a knife edge at a distant of 3 cm from the plane end when a mass of 2.5 g is hung from this end. Calculate the distance of the centre of gravity of the pencil from this plane end.
4. Two boys of weigh 400 N and 700N sit at the end of a seesaw 4 m long pivoted at the centre. What will be the position of a third boy whose weight is 600 N in order to balance the seesaw?
5. When a mass of 50 g is hung from at the 5 cm marked of a uniform metre rule, the rule balances on a knife edge place at the 35cm mark. What is the weight of the metre rule?
6. Differentiate between density and relative density.

READING ASSIGNMENT

Read up Simple Harmonic Motion in New School Physics by M A Anyakoha and answer the following questions:

1. what is Simple Harmonic Motion
2. Mention four examples of bodies in SHM

WEEK 9:

DATE

TOPIC; SIMPLE HARMONIC MOTION

CONTENT

- CONCEPT of S.H.M
- Example of bodies in S.H.M
- Mathematical description of SHM
- Terms used in describing SHM
- Energy conversion in SHM
- Damped oscillation
- Forced vibration in SHM
- Resonance

PERIOD ONE

CONCEPT OF SHM

Simple harmonic motion is an example of periodic motion. A periodic motion is one whose pattern of motion is repeated at regular interval of time.

A body is said to be in Simple Harmonic Motion if it moves along a fixed path such that its acceleration is directly proportional to its displacement from a fixed point and it is also directed toward that fixed point.

Simple harmonic motion can be defined as the motion of a body whose acceleration is always directed towards a fixed point and is proportional to the displacement of the bodies from that point.

Example of bodies in SHM

1. A vibrating simple pendulum
2. A mass at the end of a vibrating helical spring
3. Oscillation of mercury in a U-tube
4. Motion of the balance wheel of a watch.
5. Motion of prongs of a vibrating tuning fork
6. Motion of a loaded test tube in water
7. Motion of a normal human heart beat
8. Motion of a see-saw

9. Motion of a cantilever

Mathematical definition of SHM

Mathematically, SHM can be defining as;

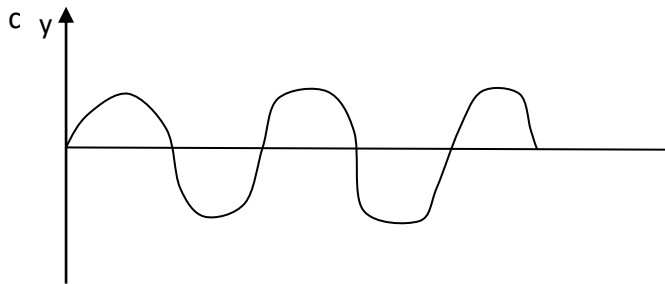
$$a \propto -y$$

Where a is the acceleration and y is the displacement.

Introducing a constant,

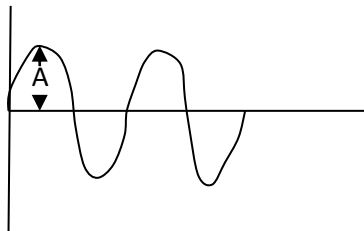
$$a = -\omega^2 y$$

The displacement of bodies in SHM simulate the sinusoidal change describe by the sine



Terms for describing a SHM

1. Amplitude (A). this the maximum displacement from the equilibrium position.



2. Period (T); this is the time taken for the body in SHM to complete one oscillation. If a body in SHM complete n cycles/oscillations in time t, the period of the SHM is given as.

$$\text{period} \quad T = \frac{t}{n} \dots \dots \dots (i)$$

The S.I unit of period is seconds

3. Frequency (f); this is the number of cycles completed by a body in SHM in one seconds. The S.I unit of frequency is Hertz (s^{-1}). Frequency can also be defined as the reciprocal of period,

$$\begin{aligned} \text{frequency} \quad f &= \frac{1}{T} \dots \dots \dots (ii) \\ f &= \frac{n}{t} \end{aligned}$$

4. Angular frequency(ω); this is the ratio of one complete cycle to the period of the SHM. Angular frequency is sometime referred to as angular speed. It S.I unit is rad/s. another unit for ω is rev/min

$$\text{angular frequency} \quad \omega = \frac{2\pi}{T}$$

$$\omega = 2\pi f \dots \dots \dots (iii)$$

5. Displacement (y); the displacement of a body in SHM simulate the sine curve and it is given as;

$$y = A \sin \theta$$

θ is the angular displacement and it is given as $\theta = \omega t$
t)

$$y = A \sin \omega t \dots \dots \dots (iv)$$

Where y is the displacement, A is the amplitude, ω is the angular frequency and t is time.

EVALUATION

1. State five example of bodies in SHM
2. Define the following terms (i) amplitude (ii) frequency (iii) period

PERIOD TWO.

SPEED AND ACCELERATION OF BODIES IN SHM

6. Speed (v); this is the rate of change of displacement/distance. Since this is not a uniform motion, we result to differential calculus

$$v = \frac{\partial y}{\partial t}$$

$$v = \frac{\partial (A \sin \omega t)}{\partial t}$$

$$V = A \omega \cos \omega t \dots \dots \dots (v)$$

Speed of a body in SHM can also be given as,

$$v = \omega \sqrt{A^2 - Y^2} \dots \dots \dots (vb)$$

A body in SHM will have it maximum value when $\sin \omega t = 1$

$$\text{maximum speed} \quad V_{max} = \omega A \dots \dots \dots (vi)$$

7. Acceleration (a); this is the rate of change of velocity.

$$a = \frac{\partial V}{\partial t}$$

$$a = \frac{\partial (A \omega \cos \omega t)}{\partial t}$$

$$a = -A \omega^2 \sin \omega t \dots \dots \dots (v)$$

(students who had not taken lesson in differential calculus should not bother about the derivation. But they should take note of the result)

$$a_{max} = A \omega^2$$

maximum acceleration occur when angular displacement $\theta = \frac{\pi}{2}$ and $\sin \theta = 1$.

EXPERIMENT

- To determine acceleration due to gravity g using simple pendulum.
- To determine the force constant of a helical spring

CLASSWORK

1. A body executing simple harmonic motion has an angular velocity of 22rad s^{-1} . If it has a maximum displacement of 10cm . what is its maximum linear velocity?

$$\omega = 22\text{rad s}^{-1}$$

$$A = 10\text{cm} = 0.1\text{m}$$

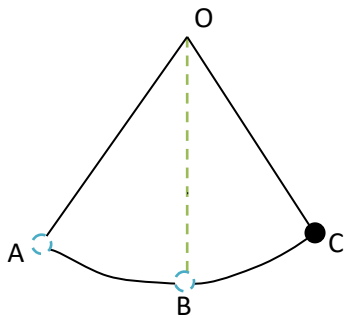
$$v = \omega A$$

$$V = 22 \times 0.1$$

$$V = 2.2\text{ms}^{-1}$$

Simple pendulum

This consists of a small mass attached to the end of a string.



B is the equilibrium position. As the body passes through this position its kinetic energy is maximum.

At B, speed is maximum kinetic energy is maximum potential energy is zero

At A and C, the body is temporarily at rest. Speed is zero as well as kinetic energy. But the potential energy at this point is maximum.

The period of oscillation of a simple pendulum

1. Period is directly proportional to the square root of the length of the string

$$T \propto \sqrt{l}$$

Period increases with length.

$$\frac{T_1}{T_2} = \sqrt{\frac{l_1}{l_2}}$$

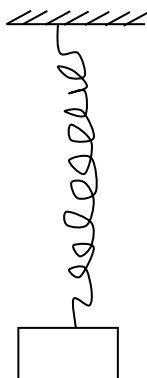
2. Is independent on the mass of the bob
3. Is inversely proportional to the square root of the acceleration due to gravity.

$$T \propto \frac{1}{\sqrt{g}}$$

$$T = 2\pi \sqrt{\frac{l}{g}} \quad \dots \dots \dots (v)$$

A body at the end of a vibrating helical spring.

This is another example of a simple harmonic motion. It consists of a mass attached to the end of a spring.



The period of the mass vibrating at the end of the helical spring is

1. Directly proportional to the square root of the mass

$$T \propto \sqrt{m}$$

Period increases with mass

$$\frac{T_1}{T_2} = \sqrt{\frac{m_1}{m_2}}$$

2. Inversely proportional to the square root of the force constant of the spring.

$$T \propto \frac{1}{\sqrt{k}}$$

The period of a vibrating mass at the end of a helical spring is given as

$$T = 2\pi \sqrt{\frac{m}{k}}$$

EVALUATION.

1. The number of cycle per seconds completed by a body in SHM is called ---- (a) period (b) amplitude (c) angular frequency (d) frequency
2. Which of these is not true about bodies in SHM? (a) acceleration is proportional to displacement (b) acceleration act in opposite direction to the displacement (c) acceleration is directed toward the fixed point

3. A simple pendulum has a period of 4.2 s. when the length is shortened by 1m, the period is 3.7 s. calculate the original length of the string. 9a) 74.5 m (b) 3.2 m (c) 2.7 m (d) 1.8 m (ACEDEX, 2011)

Period 3: ENERGY IN SHM

A body in simple harmonic motion undergoes displacement as a result of a restoring force acting on its toward the equilibrium position. Energy is always involved when a body moves through a distance under the action of a force.

Recall work done = force x distance

Consider a vibrating mass at the end of a helical spring; if an average force of $\frac{1}{2}F$ act on the mass to cause a displacement of y

Work done = energy = average force x distance

$$E = \frac{1}{2} F \times y$$

$$E = \frac{1}{2} Fy \dots \dots \dots (vi)$$

This is equivalent to the elastic potential energy stored in the spring.

But $F = Ky$

Where k - force constant or elasticity constant y - extension/ displacement

Substituting this into (vi)

$$E = \frac{1}{2} (ky)y = \frac{1}{2} ky^2$$

$$E = \frac{1}{2} ky^2 \dots \dots \dots (vii)$$

The potential energy of the mass is maximum when y = amplitude

$$E = PE_{max} = \frac{1}{2} kA^2 \dots \dots \dots (viii)$$

The kinetic energy of the mass is given as

$$KE = \frac{1}{2} mv^2$$

But from eqn (vb)

$$v = \omega \sqrt{A^2 - y^2}$$

Therefore, KE at displacement y is given as

$$KE = \frac{1}{2} m \omega^2 (A^2 - y^2) \dots \dots \dots (ix)$$

Maximum kinetic energy will occur at the equilibrium position (i.e when $y = 0$)

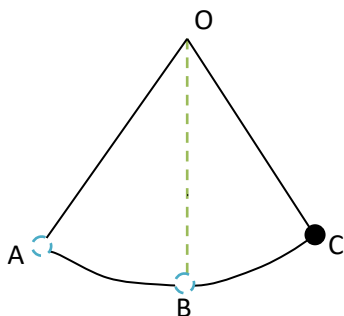
$$KE_{max} = \frac{1}{2} m \omega^2 A^2 \dots \dots \dots (x)$$

EVALAUTION

1. What is the angular speed of a body vibrating at 50cycles per second? (a) $200\pi \text{ rads}^{-1}$ (b) $100\pi \text{ rads}^{-1}$ (b) 50 rads^{-1} (d) 0.01 rads^{-1}
2. If a body moving with SHM has an angular velocity of 50 rad/s and amplitude 10 cm , calculate it linear velocity.
3. A body in SHM has an amplitude of 10 cm and a frequency of 100 Hz calculate (i) acceleration at maximum displacement (b) period of oscillation (c) velocity at the centre of the motion

PERIOD 4

Simple pendulum



The energy of the bob at B is entirely kinetic (equilibrium position)

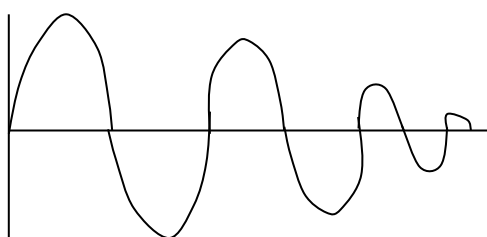
The energy of the bob at A and C is entirely potential (the bob is temporarily at rest at these points).

At any point between A and B or C and B, the energy is the sum of the potential and the kinetic energy of the bob at that position.

Damped oscillation

SHM is an hypothetical motion in which energy has been taking to be constant through the motion and the amplitude does not change. However, a real-life situation is the damp oscillation in which amplitude die out with time due to air resistance. Energy of the system also depreciate with time.

In damped harmonic oscillation, the amplitude decreases with time until it is zero.



The amplitude of this motion is gradually decreasing

To maintain an oscillation that would have been damped in simple harmonic motion, an external periodic force is applied. This is called **forced vibration**

Forced vibration is a vibration resulting from the action of an external periodic force on an oscillating body.

Resonance; this is a phenomenon in which the frequency of the external oscillator coincides with the natural frequency of a body thereby making the body to vibrate with a large amplitude.

Resonance explains why sometime at a radio playing some tunes could make a tumbler on the same table to shake visibly.

EVALUATION

1. A simple pendulum has a period of 3.0 s. If the value of $g = 9.9 \text{ m/s}^2$. Calculate the length of the pendulum.
2. An object moving with SHM has amplitude 5 cm and frequency 50Hz. Calculate (i) period of the oscillation (ii) acceleration at the middle and end of the oscillation (iii) velocity at the middle and at the end of the oscillation
3. Define the following (i) damped oscillation (ii) forced oscillation (iii) resonance

WEEK END ASSIGNMENT

1. Sketch the curve for displacement speed and acceleration and state the phase difference between them.
2. Beginning from $V = A \omega \cos \omega t$ show that V

$$V = \omega \sqrt{A^2 - y^2}$$

3. Describe an experiment to verify the variation of the period of a simple pendulum with length of the pendulum.
4. The period of a simple pendulum is 3.45 s. when the length of the pendulum is shortened by 1 m, the period is 2.81 s. calculate (a) the original length of the pendulum (b) the acceleration due to gravity

READING ASSIGNMENT

Students should read up Simple Harmonic Motion in their text books.

WEEK 10: REVISION

WEEK 11: EXAMINATION