SS 2 MATHEMATICS

A. Term Overview and Key Concepts

FIRST TERM MATHEMATICS TOPICS:

WEEK 1: REVISION
LOGARITHMS
├─ Log Tables
238239
├─ Antilogarithms
└─ Calculations
WEEK 2: ALGEBRAIC FRACTIONS
└─ Algebra, Fractions, Equations
WEEK 3: NUMBERS & APPROXIMATION
├─ Standard Form
├─ Significant Figures
└─ Percentage Error
WEEK 4: SEQUENCES & SERIES
├─ Arithmetic Progression
├─ nth Term Formula
└─ Sum of Series
WEEK 5: SEQUENCE AND SERIES
├─ Geometric Progression
├─ nth Term Formula
└─ Sum of G.P.
WEEK 6: QUADRATIC EQUATIONS I
├─ Factorization
├─ Completing Square
└─ Quadratic Formula
WEEK 7: QUADRATIC EQUATIONS II
├─ Forming Equations
├─ Sum & Product of Roots
└─ Word Problems
WEEK 8: SIMULTANEOUS EQUATIONS
├─ Elimination Method
├─ Substitution Method
└─ Linear-Quadratic
WEEK 9: GRAPHICAL SOLUTIONS
├─ Linear Graphs
├─ Parabolas
└─ Intersection Points

WEEK 1: USE OF LOGARITHM TABLES AND CALCULATORS

Learning Objectives:

By the end of the lesson, students should be able to:

1. Define logarithms and state their properties
2. Use logarithm tables to find logarithms and antilogarithms
3. Perform multiplication, division, powers, and roots using logarithms
4. Use scientific calculators for logarithmic calculations
5. Apply logarithms to solve practical problems

Entry Behaviour:

Students understand indices and can perform basic operations with powers.

Instructional Materials:

- Logarithm tables (base 10)
- Antilogarithm tables
- Scientific calculators
- Charts showing logarithm properties
- Whiteboard and markers
- Practice worksheets

Content:

A. Introduction to Logarithms

Definition: A logarithm is the power to which a base must be raised to produce a given number.

Diagram 46: The Logarithm Concept

```
FUNDAMENTAL RELATIONSHIP:
If: bˣ = y
Then: log_b (y) = x

"Logarithm is the inverse of exponentiation"
VISUAL MODEL:
EXPONENTIATION
───────────────→
Base ^Power = Number
b ^x = y
←───────────────
LOGARITHM
log_b(y) = x

EXAMPLES:
1. 10² = 100
↓
log₁₀(100) = 2
Meaning: "What power of 10 gives 100?"
Answer: 2
2. 2³ = 8
↓
log₂(8) = 3
Meaning: "What power of 2 gives 8?"
Answer: 3
```

3. $10^0 = 1$
↓
$\log_{10}(1) = 0$
Meaning: "What power of 10 gives 1?"
Answer: 0

Common Logarithm (Base 10):

Diagram 47: Understanding Common Logarithms

NOTATION:
$\log_{10}(x) = \log(x)$
(Base 10 is usually omitted)
POWERS OF 10:
Power
Number
Logarithm

$10^0 = 1$
→ log(1) = 0
$10^1 = 10$
→ log(10) = 1
$10^2 = 100$
→ log(100) = 2
$10^3 = 1{,}000$
→ log(1,000) = 3
$10^4 = 10{,}000$
→ log(10,000) = 4
PATTERN:
Each time number ×10, log increases by 1
Example:
50 is between 10 and 100
So log(50) is between 1 and 2
Actually: log(50) ≈ 1.699

B. Laws of Logarithms

Diagram 48: The Five Fundamental Laws

LAW 1: PRODUCT RULE

$$\boxed{\log(M \times N) = \log M + \log N}$$

Visual proof:
If $M = 10^a$ and $N = 10^b$
Then $M \times N = 10^a \times 10^b = 10^{(a+b)}$
So $\log(M \times N) = a + b = \log M + \log N$
Example:
log(100 × 1000) = log 100 + log 1000
= 2 + 3 = 5
Check: $100 \times 1000 = 100{,}000 = 10^5$ ✓

LAW 2: QUOTIENT RULE

$$\boxed{\log(M \div N) = \log M - \log N}$$

Visual:

$$\frac{M}{N} \text{ means "divide"}$$

$\log(M/N) = \log M - \log N$

Example:

$\log(1000 \div 10) = \log 1000 - \log 10$

$= 3 - 1 = 2$

Check: $1000 \div 10 = 100 = 10^2$ ✓

LAW 3: POWER RULE

$$\boxed{\log(M^n) = n \times \log M}$$

Visual:

$M^3 = M \times M \times M$

$\log(M^3) = \log(M \times M \times M)$

$= \log M + \log M + \log M$

$= 3 \log M$

Example:58

$\log(10^3) = 3 \times \log 10$

$= 3 \times 1 = 3$

Check: $10^3 = 1000$, $\log 1000 = 3$ ✓

LAW 4: ROOT RULE

$$\boxed{\log(\sqrt[n]{M}) = (1/n) \times \log M}$$

Since $\sqrt[n]{M} = M^{(1/n)}$

$\log(\sqrt[n]{M}) = \log(M^{(1/n)}) = (1/n) \log M$

Example:

$\log(\sqrt{100}) = \log(100^{(1/2)})$

$= (1/2) \times \log 100$

$= (1/2) \times 2 = 1$

Check: $\sqrt{100} = 10$, $\log 10 = 1$ ✓

LAW 5: SPECIAL VALUES

log(1) = 0

log(10) = 1

$\log(10^n) = n$

Why log(1) = 0:
$10^0 = 1$, so $\log_{10}(1) = 0$
SUMMARY DIAGRAM:
Multiply → Add logs
Divide → Subtract logs
Power → Multiply log
Root → Divide log

C. Structure of Logarithm Tables

Diagram 49: Logarithm Table Anatomy PARTS OF A LOGARITHM:

```
log 456 = 2.6590
↑ ↑
│ └─ MANTISSA (decimal part)
└─── CHARACTERISTIC (integer part)
```

CHARACTERISTIC:
The integer part of the logarithm
Depends on position of decimal point
For numbers ≥ 1:
Characteristic = (number of digits before decimal) - 1
Examples:
456 → 3 digits → Characteristic = 2
4.56 → 1 digit → Characteristic = 0
45600 → 5 digits → Characteristic = 4
For numbers < 1:
Characteristic = -(number of zeros after decimal + 1)
Written with bar notation: $\bar{1}$, $\bar{2}$, $\bar{3}$
Examples:
0.456 → no zeros → Characteristic = -1 = $\bar{1}$
0.0456 → one zero → Characteristic = -2 = $\bar{2}$
0.00456 → two zeros → Characteristic = -3 = $\bar{3}$
MANTISSA:
The decimal part (from tables)
Always positive
Found by looking up first few digits

5960
LOGARITHM TABLE LAYOUT:
Mean Difference Columns
0 1 2 3 4 5 6 7 8 9

45	65	65	65	66	66	66	66	66	67	67
46	66	66	67	67	67	67	68	68	68	68
47	67	67	67	68	68	68	68	69	69	69

↑
↑
First 2
Third digit
digits
(column)
To find log 4.56:
1. Find row 45
2. Look in column 6
3. Read mantissa: 0.6590
USING MEAN DIFFERENCE:
For more precision (4th digit)
To find log 4.567:
1. Find log 4.56 = 0.6590
2. Look in mean difference column 7
3. Add the difference
4. Final mantissa: 0.6590 + small adjustment

D. Using Logarithm Tables - Step by Step

Diagram 50: Finding Logarithms

EXAMPLE 1: Find log 456
STEP 1: Express in standard form

$456 = 4.56 \times 10^2$

↓
↓

Mantissa Char

STEP 2: Find characteristic
Number of digits = 3
Characteristic = 3 - 1 = 2
STEP 3: Find mantissa from table
Look up 4.56:
Row 45, Column 6

$\boxed{0.6590}$

STEP 4: Combine

log 456 = 2.6590

↑
↑

Char Mantissa

VISUAL CHECK:
100 < 456 < 1000
↓
↓
log 100 < log 456 < log 1000
2
< 2.6590 <
3
✓

EXAMPLE 2: Find log 0.0342
STEP 1: Express in standard form

$0.0342 = 3.42 \times 10^{-2}$

↓
↓

Mantissa
Char

STEP 2: Find characteristic62
One zero after decimal
Characteristic = -(1 + 1) = -2
Written as $\bar{2}$
STEP 3: Find mantissa
Look up 3.42:
Row 34, Column 2

$\boxed{0.5340}$

STEP 4: Combine

```
┌─────────────────────────┐
│ log 0.0342 = 2̄.5340
│
↑
↑
│
Char Mantissa│
└─────────────────────────┘
```

MEANING: $\bar{2}.5340 = -2 + 0.5340$

PRACTICE GUIDE:
Number
Standard Form
Char Mantissa

567
5.67×10^2
2
0.7536
56.7
5.67×10^1
1
0.7536
5.67
5.67×10^0
0
0.7536
0.567
5.67×10^{-1}
$\bar{1}$
0.7536
0.0567
5.67×10^{-2}
$\bar{2}$
0.7536
Notice: Same mantissa for same digits!

E. Finding Antilogarithms

Diagram 51: Antilogarithm Process

ANTILOGARITHM = Inverse of logarithm
If log x = y, then x = antilog y
EXAMPLE: Find antilog 2.6590
STEP 1: Separate parts

```
┌─────────────────────────┐
│ 2.6590
```

```
│
│ ↑
↑
│
│ │
└─ Mantissa: 0.6590│
│ └────── Characteristic:2│
└─────────────────────────┘
```

STEP 2: Find number from mantissa
Look up 0.6590 in antilog table:
Or reverse lookup in log table
0.6590 → 4.56
STEP 3: Apply characteristic
Characteristic = 2 means × 10^2
Number = 4.56 × 10^2
= 456
ANSWER: antilog 2.6590 = 456

ANTILOG TABLE STRUCTURE:

Mantissa	0	1	2	3
.659	456	457	457	458
.660	457	458	458	459

To find antilog 2.6590:
1. Look for mantissa .659
2. Column 0 gives 456
3. Apply characteristic: 4.56 × 10^2 = 456

NEGATIVE CHARACTERISTICS:
Find antilog 2.5340
STEP 1: Mantissa lookup
0.5340 → 3.42
STEP 2: Apply characteristic
$\bar{2}$ means × 10^{-2}
Number = 3.42 × 10^{-2}
= 0.0342
Bar notation guide:
$\bar{1}$ × 10^{-1} → move decimal 1 left
$\bar{2}$ × 10^{-2} → move decimal 2 left

3→ $\times 10^{-3}$ → move decimal 3 left

F. Calculations Using Logarithms

1. Multiplication:

Diagram 52: Multiplication with Logs

PROBLEM: Calculate 34.5×2.76
STEP 1: Set up equation
Let $N = 34.5 \times 2.76$
STEP 2: Take log of both sides
$\log N = \log(34.5 \times 2.76)$
STEP 3: Apply product rule
$\log N = \log 34.5 + \log 2.76$
STEP 4: Find individual logs

```
log 34.5:

34.5 = 3.45 × 10^1

Char = 1, Mantissa from
table = 0.5378
```

65

```
log 34.5 = 1.5378
```

```
log 2.76:

2.76 = 2.76 × 10^0

Char = 0, Mantissa from
table = 0.4409

log 2.76 = 0.4409
```

STEP 5: Add the logs
1.5378
+ 0.4409
———————
1.9787
STEP 6: Find antilog
$\log N = 1.9787$
Mantissa 0.9787 → 9.52 (from table)
Characteristic 1 → $\times 10^1$
$N = 9.52 \times 10 = 95.2$
ANSWER: $34.5 \times 2.76 \approx 95.2$
VISUAL PROCESS:

```
┌───────────────────────┐
  34.5 × 2.76
│
↓
│
│ log 34.5 + log 2.76 │
  1.5378 + 0.4409
│
↓
│
1.9787
│
↓
│
  antilog 1.9787
│
↓
│
95.2
└───────────────────────┘
```

[NOTE: Check your textbook for proper arrangements]

2. Division:

Diagram 53: Division with Logs

```
PROBLEM: Calculate 845 ÷ 23.4
STEP 1: Set up
Let N = 845 ÷ 23.4
STEP 2: Apply quotient rule
log N = log 845 - log 23.4
STEP 3: Find individual logs
log 845 = 2.9269
log 23.4 = 1.3692
STEP 4: Subtract
2.9269
- 1.3692
─────────
1.5577
STEP 5: Find antilog
Mantissa 0.5577 → 3.61
```

Characteristic 1 → × 10^1
N = 36.1
ANSWER: 845 ÷ 23.4 ≈ 36.1
COMPUTATION TABLE:

Number	Log	Operation
845	2.9269	(given)
23.4	1.3692	(given)
	————————	
Result	1.5577	(subtract)
Antilog	36.1	(answer)

3. Powers:

Diagram 54: Finding Powers with Logs PROBLEM: Calculate $(3.25)^4$
STEP 1: Set up
Let N = $(3.25)^4$
STEP 2: Apply power rule
log N = 4 × log 3.25
STEP 3: Find log 3.25
log 3.25 = 0.5119
STEP 4: Multiply by power
log N = 4 × 0.5119
= 2.0476
STEP 5: Find antilog
Mantissa 0.0476 → 1.116
Characteristic 2 → × 10^2
N = 1.116 × 10^2 = 111.6
ANSWER: $(3.25)^4$ ≈ 111.6
VISUAL:
$(3.25)^4$
↓
4 × log 3.25
4 × 0.5119
↓
2.0476

```
↓
antilog 2.0476
↓
111.6
VERIFICATION:
3.25 × 3.25 = 10.5625
10.5625 × 3.25 = 34.328125
34.328125 × 3.25 ≈ 111.6 ✓
```

4. Roots:

67**Diagram 55: Finding Roots with Logs**

```
PROBLEM: Calculate ∛578
STEP 1: Set up
Let N = ∛578 = 578^(1/3)
STEP 2: Apply root rule
log N = (1/3) × log 578
STEP 3: Find log 578
log 578 = 2.7619
STEP 4: Divide by root index
log N = 2.7619 ÷ 3
= 0.9206
STEP 5: Find antilog
Mantissa 0.9206 → 8.33
Characteristic 0 → × 10⁰
N = 8.33
ANSWER: ∛578 ≈ 8.33
DETAILED DIVISION:
0.9206
┌────────
3 │2.7619
 │2.7
 │──────
 │ 0.0619
 │ 0.0600
 │──────
 │ 0.0019 (remainder)
ROOT TYPES:
√x = x^(1/2) → log N = (1/2) log x
∛x = x^(1/3) → log N = (1/3) log x
⁴√x = x^(1/4) → log N = (1/4) log x
```

5. Complex Expressions:

Diagram 56: Combined Operations

```
PROBLEM: (45.6 × 0.235) ÷ √12.8
STEP 1: Set up
Let N = (45.6 × 0.235) ÷ √12.8
STEP 2: Apply log laws
```

log N = log (45.6 × 0.235) - log (√12.8)
log N = log 45.6 + log 0.235 - (1/2) log 12.8
STEP 3: Find individual logs

log 45.6 = 1.6590

log 0.235 = $\bar{1}$.3711

= -0.6289

log 12.8 = 1.1072

STEP 4: Substitute and calculate
log N = 1.6590 + (-0.6289) - (0.5) (1.1072)

= 0.4765
STEP 5: Find antilog
CALCULATION TABLE:

Term	Log	Sign
45.6	1.6590	
	-0.6289	+
√12.8	0.5536	-
Total	0.4765	
Antilog 2.995		

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G. Using Scientific Calculators

Diagram 57: Calculator Functions

SCIENTIFIC CALCULATOR LAYOUT:

[DISPLAY]

[LOG] [10^x] [LN] [e^x]

[7][8][9] [÷] [×]

[4][5][6] [-] [+]

[1][2][3] [=]

[0] [.] [±] [SHIFT]

KEY FUNCTIONS:
[LOG] - Calculates $\log_{10}(x)$
Example: 100 [LOG] = 2
[10^x] or [SHIFT][LOG] - Antilog
Example: 2 [SHIFT][LOG] = 100
[LN] - Natural log (base e)
Example: 10 [LN] ≈ 2.303
[e^x] or [SHIFT][LN] - Inverse of ln
Example: 1 [SHIFT][LN] ≈ 2.718

CALCULATOR STEPS:
MULTIPLICATION: 34.5 × 2.76
Traditional: 34.5 [×] 2.76 [=]
Result: 95.22Using logs:
34.5 [LOG] [+] 2.76 [LOG] [=]
Result: 1.9787
[SHIFT] [LOG]
Result: 95.22
POWER: $(3.25)^4$
Traditional: 3.25 [^] 4 [=]
Result: 111.601...
Using logs:
4 [×] 3.25 [LOG] [=]
Result: 2.0476
[SHIFT] [LOG]
Result: 111.6
ROOT: $\sqrt[3]{578}$

Traditional: 578 [SHIFT] [∛] 3 [=]
Or: 578 [^] [(] 1 [÷] 3 [)] [=]
Result: 8.33
Using logs:
578 [LOG] [÷] 3 [=]
Result: 0.9206
[SHIFT] [LOG]
Result: 8.33

ADVANTAGES OF CALCULATOR:
✓ Faster calculations
✓ More precise results
✓ Can handle complex expressions
✓ Less prone to reading errors
ADVANTAGES OF LOG TABLES:
✓ Understanding log properties
✓ Exam situations without calculator
✓ Verification of calculator results
7172

Worked Examples with Detailed Solutions:

Example 1: Use logarithm tables to find log 237

Diagram 58: Complete Solution

Given: 237
STEP 1: Express in standard form
$237 = 2.37 \times 10^2$
Diagram:
237
↓
2.37×10^2
↑
↑
(1≤A<10) (power)
STEP 2: Find characteristic
Number of digits = 3
Characteristic = 3 - 1 = 2
Visual:
237 = H T U . tenths
2 3 7 .
↑ ↑ ↑
3 digits
Characteristic = 3 - 1 = 2
STEP 3: Find mantissa
Look up 2.37 in log table:
Row 23, Column 7
Table excerpt:

```
┌──────┬──────┬──────┬──────┬──────┬──────┬──────┬──────┐
23  │362│365│367│370│372│375│377│380│
└──────┴──────┴──────┴──────┴──────┴──────┴──────┴──────┘
```

↑
Mantissa = 0.3747STEP 4: Combine
log 237 = Characteristic + Mantissa
= 2 + 0.3747
= 2.3747
VERIFICATION:
100 < 237 < 1000
log 100 < log 237 < log 1000
2 < 2.3747 < 3 ✓
ANSWER: log 237 = 2.3747
ALTERNATIVE METHOD (Calculator):
237 [LOG] = 2.374748...
Rounded: 2.3747 ✓

Example 2: Calculate 52.3 × 8.94 using logarithms

Diagram 59: Multiplication Solution

Given: 52.3 × 8.94
STEP 1: Set up
Let N = 52.3 × 8.94
log N = log 52.3 + log 8.94
STEP 2: Find log 52.3
52.3 = 5.23 × 10^1
Characteristic = 1
Table lookup for 5.23:
Row 52, Column 3
Mantissa = 0.7185
log 52.3 = 1.7185
STEP 3: Find log 8.94
8.94 = 8.94 × 10^0
Characteristic = 0
7374
Table lookup for 8.94:
Row 89, Column 4
Mantissa = 0.9513
log 8.94 = 0.9513
STEP 4: Add logs

```
┌──────────────────┐
│
1.7185
│
│ + 0.9513
│
│ ________
```

2.6698

STEP 5: Find antilog
log N = 2.6698
Characteristic = 2 → × 10^2
Mantissa 0.6698 → 4.675 (from antilog table)
N = 4.675 × 10^2
= 467.5
ANSWER: 52.3 × 8.94 ≈ 467.5
VERIFICATION (Calculator):
52.3 × 8.94 = 467.562
Rounded: 467.5 ✓
ERROR ANALYSIS:
Actual: 467.562
Approximation: 467.5
Error: 0.062
% Error: 0.01% (very accurate!)
WORKING TABLE:

Number Standard Form Log

52.3
5.23 × 10^1
1.7185
75
8.94
8.94 × 10^0
0.9513

Sum of logs:
2.6698

Antilog:
467.5

Example 3: Find $(2.45)^5$ using logarithm tables
Diagram 60: Power Calculation
Given: $(2.45)^5$
STEP 1: Set up

Let $N = (2.45)^5$

$\log N = 5 \times \log 2.45$

Visual:

$(2.45)^5$

↓

$2.45 \times 2.45 \times 2.45 \times 2.45 \times 2.45$

↓

5 times multiplication

↓

$5 \times \log 2.45$

STEP 2: Find log 2.45

$2.45 = 2.45 \times 10^0$

Characteristic = 0

Table lookup:

Row 24, Column 5

Mantissa = 0.3892

log 2.45 = 0.3892

STEP 3: Multiply by power

0.3892

×

5

1.9460

76

Calculation detail:

0.3892

×

5

1.9460

STEP 4: Find antilog

log N = 1.9460

Characteristic = 1 → $\times 10^1$

Mantissa 0.9460 → 8.83 (from table)

$N = 8.83 \times 10^1$

= 88.3

ANSWER: $(2.45)^5 \approx 88.3$

VERIFICATION STEPS:

$2.45^2 = 6.0025$

$2.45^3 = 14.706...$
$2.45^4 = 36.03...$
$2.45^5 = 88.26... \approx 88.3$ ✓
POWER TABLE:

Power	Value
2.45^1	2.45
2.45^2	6.00
2.45^3	14.71
2.45^4	36.03
2.45^5	88.26

Example 4: Calculate $\sqrt[4]{2560}$ using logarithms
Diagram 61: Fourth Root Solution77
Given: $\sqrt[4]{2560}$
STEP 1: Express as power
$\sqrt[4]{2560} = 2560^{(1/4)}$
Let $N = 2560^{(1/4)}$
$\log N = (1/4) \times \log 2560$
Visual:
$\sqrt[4]{2560}$
↓
$2560^{(1/4)}$
↓
$(1/4) \times \log 2560$
STEP 2: Find log 2560
$2560 = 2.560 \times 10^3$
Characteristic = 3
Table lookup for 2.56:
Row 25, Column 6
Mantissa = 0.4082
log 2560 = 3.4082
STEP 3: Divide by 4

3.4082 ÷ 4

↓

0.85205

Long division:

```
   0.85205
   ┌────────
4  │3.40820
   │3.2
   │────
   │0.208
   │0.20078
   │────
   │0.0082
   │0.0080
   │────
   │0.00020
```

log N = 0.8521 (rounded)

STEP 4: Find antilog

Characteristic = 0 → $\times 10^0$

Mantissa 0.8521 → 7.11 (from table)

N = 7.11×10^0

= 7.11

ANSWER: $\sqrt[4]{2560} \approx 7.11$

VERIFICATION:

7.11^4 = ?

$7.11^2 = 50.5521$

$50.5521^2 = 2555.5... \approx 2560$ ✓

ROOT COMPARISON:

Root Type	Result
$\sqrt{2560}$	50.60
$\sqrt[3]{2560}$	13.68

│ $\sqrt[4]{2560}$
7.11
│
│ $\sqrt[5]{2560}$
4.79
│
└──────────────────────────────┘

Example 5: Simplify $(0.456)^3 \times \sqrt{89.2}$ using logarithms

Diagram 62: Complex Expression

Given: $(0.456)^3 \times \sqrt{89.2}$
STEP 1: Set up
Let $N = (0.456)^3 \times \sqrt{89.2}$
$\log N = 3\log(0.456) + (1/2)\log(89.2)$79
STEP 2: Find log 0.456
$0.456 = 4.56 \times 10^{-1}$
Characteristic = $\bar{1}$ = -1
Table lookup for 4.56:
Mantissa = 0.6590
log 0.456 = $\bar{1}.6590$
= -1 + 0.6590
= -0.3410
STEP 3: Find log 89.2
$89.2 = 8.92 \times 10^1$
Characteristic = 1
Table lookup for 8.92:
Mantissa = 0.9504
log 89.2 = 1.9504
STEP 4: Apply operations
3 × log(0.456):
3 × (-0.3410) = -1.0230
(1/2) × log(89.2):
(1/2) × 1.9504 = 0.9752
STEP 5: Add results

┌──────────────────┐
-1.0230

+ 0.9752

────────

-0.0478
└──────────────────┘

log N = -0.0478
= 1.9522 (in bar notation)
STEP 6: Find antilog80

Characteristic = $\bar{1}$ × 10^{-1}
Mantissa 0.9522 → 8.96
N = 8.96 × 10^{-1}
= 0.896
ANSWER: $(0.456)^3 \times \sqrt{89.2} \approx 0.896$
STEP-BY-STEP TABLE:

Expression	Log	Operation
0.456	-0.3410	×3
$(0.456)^3$	-1.0230	
89.2	1.9504	÷2
$\sqrt{89.2}$	0.9752	
	————	+
Total	-0.0478	
Antilog	0.896	

VERIFICATION (Calculator):
$(0.456)^3$ = 0.0949...
$\sqrt{89.2}$ = 9.444...
0.0949 × 9.444 ≈ 0.896 ✓

Class Activity:

Activity 1: Log Table Treasure Hunt

Diagram 63: Practice Exercise

Find the following using log tables:
1. log 345

Answer:
2.5378

2. log 0.0678

Answer:

81

$\bar{2}.8312$

3. log 7890

Answer:

3.8971

4. antilog 1.5263

Answer:

33.6

5. antilog 3.7520

Answer:

0.00565

Activity 2: Calculator vs Tables Race

Compare results using both methods for:

- 45.6 × 23.8
- 234 ÷ 5.67
- $(3.4)^3$

Activity 3: Real-World Application

Calculate compound interest using: $A = P(1 + r)^n$ Where P = ₦10,000, r = 0.08, n = 5

Evaluation Questions:

1. Use logarithm tables to find log 237.
2. Find the value of log 0.0563.
3. Use logarithms to calculate 52.3 8.94.
4. Evaluate 456 ÷ 34.7 using logarithms.
5. Find the value of $(2.45)^5$ using logarithm tables.

Assignment:

1. Find log 0.00845 using logarithm tables. Show all working including how you determined the characteristic.2. If log x = 2.3456, find the value of x using antilogarithm tables. Express your answer in standard form.
3. Use logarithms to evaluate (78.5 23.4) ÷ 45.6. Show the complete working.
4. Calculate $\sqrt[4]{2560}$ using logarithms. Verify your answer using a calculator.
5. Simplify $(0.456)^3 \sqrt{89.2}$ using logarithm tables. Compare with calculator result.

8283

WEEK 2: ALGEBRAIC FRACTIONS

A. Review of Algebraic Expressions

Definition: An algebraic expression contains variables, constants, and operations.

Diagram 1: Anatomy of an Algebraic Expression

```
3x² + 5x - 7
│  │
│
│
│  │
│
└─ Constant term (fixed number)
│  │
└─────── Linear term (first power)
│  └───────────── Coefficient (number multiplying variable)
└─────────────── Quadratic term (second power)
COMPONENTS BREAKDOWN:2
┌──────────────────────────────────────┐
│ VARIABLE (x, y, z, etc.)
│
│ Letter representing unknown value
│
│ Example: x in "3x"
│
└──────────────────────────────────────┘

┌──────────────────────────────────────┐
│ COEFFICIENT (number before variable) │
│ Multiplier of the variable
│
│ Example: 3 in "3x"
│
└──────────────────────────────────────┘

┌──────────────────────────────────────┐
│ CONSTANT (standalone number)
│
│ Fixed value without variable
│
│ Example: -7 in "3x² + 5x - 7"
│
└──────────────────────────────────────┘

┌──────────────────────────────────────┐
│ TERM (separated by + or -)
│
│ Single part of expression
│
```

│ Example: "3x²", "5x", "-7"

└───────────────────────────────────────┘

Types of Terms:

Diagram 2: Like Terms vs Unlike Terms

LIKE TERMS
(Same variable and same power - can be combined)
3x
and
5x
──
──
│
└────┬────────┘
│
Both have x¹3
2y²
and
-7y²
───
────
│
└─────┬───────┘
│
Both have y²
Visual: Like terms are like identical boxes
that can stack together

$\boxed{3x} + \boxed{5x} = \boxed{8x}$

Stack them up!
UNLIKE TERMS
(Different variables or powers - cannot be combined)
3x
and
5y
──
──
│
└────X────────┘
│
Different variables!
2x²

```
and
2x
───
──
│
│
└─────x──────┘
│
Different powers!
Visual: Unlike terms are like different shapes
that don't fit together
┌──┐ ●●●
│3x│+ ●5y● ≠ Can't combine
└──┘ ●●●4
```

Simplification Process:

Diagram 3: Step-by-Step Simplification

```
EXAMPLE: Simplify 7x + 3y - 2x + 5y
STEP 1: Identify like terms
┌──────────────────────────────┐
│ 7x + 3y - 2x + 5y
│
│ ↓
↓
↓
↓
│
│ [x terms] [y terms]
│
└──────────────────────────────┘
STEP 2: Group like terms
┌──────────────────────────────┐
│ (7x - 2x) + (3y + 5y)
│
│
↓
↓
│
│ Group x's
Group y's
│
└──────────────────────────────┘
STEP 3: Combine coefficients
x terms:
███████ (7x)
- ██
(2x)
```

[redacted]
(5x)
y terms:
[redacted]
(3y)
+ [redacted] (5y)

[redacted] (8y)
STEP 4: Write final answer

┌──────────────────────────────┐
  5x + 8y

└──────────────────────────────┘

Expanding Brackets:5
Diagram 4: Distributive Property
RULE: a(b + c) = ab + ac
EXAMPLE 1: 3(x + 4)
Visual Distribution:

┌──────┬──────┐
  x
  4

├──────┼──────┤
3 × │ 3x │ 12 │
└──────┴──────┘

↓
↓
3x + 12
Like distributing 3 gifts to 2 people:
↓ ↓
3x 12
EXAMPLE 2: -2(3x - 5)
Step 1: Multiply -2 by each term
-2 × 3x = -6x
-2 × (-5) = +10
Step 2: Write result
-6x + 10
REMEMBER: Negative × Negative = Positive!
(-) ×
(-)
=
(+)
Sign Chart:

┌────┬────┬─────┐
│ × │ + │ - │

```
├────────┼────────┼────────┤
│  +  │  +  │  -  │
├────────┼────────┼────────┤
│  -  │  -  │  +  │
└────────┴────────┴────────┘
```
6

Factorization:

Diagram 5: Common Factor Method

EXAMPLE: Factorize $6x + 9$

STEP 1: Find common factor

$6x = 2 \times 3 \times x$

$9 = 3 \times 3$

Common factor: 3

Visual:

```
6x = ●●● ●●● x
└──┬──┘
3
9 = ●●● ●●●
└──┬──┘
3
```

STEP 2: Factor out the 3

$6x + 9 = 3(2x + 3)$

↑

↑

6÷3 9÷3

CHECK: $3(2x + 3) = 6x + 9$ ✓

EXAMPLE 2: $4x^2 + 8x$

Common factor: $4x$

$4x^2 = 4x \times x$

$8x = 4x \times 2$

Result: $4x(x + 2)$

Diagram 6: Difference of Two Squares

PATTERN: $a^2 - b^2 = (a + b)(a - b)$

EXAMPLE: $x^2 - 97$

STEP 1: Identify perfect squares

$x^2 = (x)^2$

$9 = (3)^2$

STEP 2: Apply formula

$x^2 - 9 = (x + 3)(x - 3)$

↑

↑

Square roots

Visual:

```
┌────────────┐
│
│  x²
│
```

minus

9

equals

x + 3 × x - 3

MORE EXAMPLES:

- $4x^2 - 25 = (2x)^2 - (5)^2 = (2x + 5)(2x - 5)$
- $49 - y^2 = (7)^2 - (y)^2 = (7 + y)(7 - y)$
- $16a^2 - 1 = (4a)^2 - (1)^2 = (4a + 1)(4a - 1)$

B. Review of Fractions, Decimals, and Percentages

The Conversion Triangle:

Diagram 7: Three-Way Conversion

```
PERCENTAGE
/%\
/ % \
/ % \
/_% % %_\
FRACTION
⇄
DECIMAL8
```

HOW TO CONVERT:

1 FRACTION → DECIMAL
Divide top by bottom
Example: 3/4

3 ÷ 4

= 0.75

2 DECIMAL → PERCENTAGE
Multiply by 100
Example: 0.75

0.75 × 100

= 75%

3 PERCENTAGE → FRACTION
Put over 100, then simplify

Example: 75%

75/100

= 3/4 (÷25)

4 FRACTION → PERCENTAGE
Two methods:
a) Fraction → Decimal → %
b) Multiply by 100%
Example: 3/4

3/4 × 100%

= 75%

9

5 DECIMAL → FRACTION
Write as fraction, simplify
Example: 0.75

75/100

= 3/4

6 PERCENTAGE → DECIMAL
Divide by 100
Example: 75%

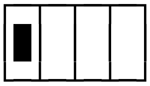
75 ÷ 100

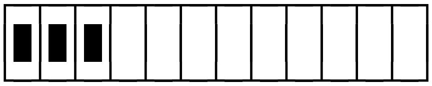
= 0.75

Fraction Operations:

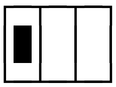
Diagram 8: Adding Fractions

RULE: Need same denominator (bottom number)
EXAMPLE: 1/4 + 1/3
STEP 1: Find common denominator
Multiples of 4: 4, 8, 12, 16...
Multiples of 3: 3, 6, 9, 12, 15...
LCD (Least Common Denominator) = 12
STEP 2: Convert both fractions
Visual representation:
1/4 = ?/12

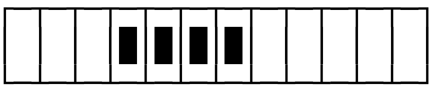
 ← 1 out of 4 parts
10
Same area in 12 parts:

 ← 3 out of 12

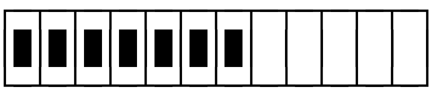
1/4 = 3/12
1/3 = ?/12

 ← 1 out of 3 parts

Same area in 12 parts:

 ← 4 out of 12

1/3 = 4/12
STEP 3: Add the numerators

 ← 7 out of 12

3/12 + 4/12 = 7/12
FORMULA:

1/4 + 1/3 = 3/12 + 4/12

= 7/12

Diagram 9: Multiplying Fractions
RULE: Multiply straight across
(numerator × numerator)
(denominator × denominator)
EXAMPLE: 2/3 × 4/5
Visual Model:
Take 4/5 of a whole:11

 4 parts shaded, 1 part not

(4 out of 5)
Now take 2/3 of that shaded area:


↓ ↓
↓

2 of the 3 rows of shaded area

Result: 8 small squares shaded
out of 15 total squares
CALCULATION:
2
×
4
= 2×4 = 8
—
—
———
——
3
5
3×5
15
Step-by-step:

2×4 = 8 (top)

3×5 = 15 (bottom)
Answer: 8/15

SHORTCUT: Cancel before multiplying
2
×
4
—
—
3
5
No common factors, so multiply:
= 8/15

Diagram 10: Dividing Fractions

RULE: Keep, Change, Flip (KCF Method)
EXAMPLE: 2/3 ÷ 4/512
STEP 1: KEEP first fraction
2/3
STEP 2: CHANGE ÷ to ×
2/3 ×
STEP 3: FLIP second fraction
2/3 × 5/4
STEP 4: Multiply

2/3 × 5/4 = 10/12 = 5/6

Visual:

```
┌────────────────────────────┐
│ 2/3 ÷ 4/5
│
│ ↓
↓
↓
│
│ KEEP CHANGE FLIP
│
│ ↓
↓
↓
│
│ 2/3 × 5/4 = 10/12 = 5/6
│
└────────────────────────────┘
```

WHY IT WORKS:
"How many 4/5's fit into 2/3?"
Think of pizza slices:
If you have 2/3 of a pizza and each
serving is 4/5 of that size, you get
5/6 of a serving.

C. Review of Linear Equations

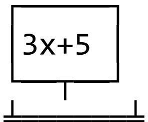
Diagram 11: Equation Balance Concept

EQUATION: 3x + 5 = 14
Think of it as a BALANCE SCALE:
Left Side
Right Side

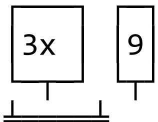
```
┌──────────┐
┌──────────┐
│ 3x + 5 │
│
14
│
└────┬─────┘
└────┬─────┘ 13
│
═════╧═══════════════╧═══════
```

Equal weight!
GOLDEN RULE:
Whatever you do to one side,
you MUST do to the other side
to keep it balanced!

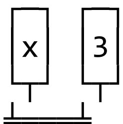
SOLVING STEPS:
Original: 3x + 5 = 14



Subtract 5: 3x = 9



Divide by 3: x = 3



BALANCED!

Diagram 12: Types of Linear Equations

TYPE 1: ONE-STEP EQUATIONS

---

x + 5 = 12
(-5) (-5)

---

x = 7
Visual:
▓▓▓▓▓ + █████ = ████████████14
x
5
12
Remove 5:
▓▓▓▓▓ = ███████
x
7

TYPE 2: TWO-STEP EQUATIONS

2x + 3 = 11
Step 1: Subtract 3
2x = 8
Step 2: Divide by 2
x = 4
Visual:
2 groups of x + 3 = 11
▓▓▓ ▓▓▓
+ █ █ █ = 11 total
Remove 3: ▓▓▓ ▓▓▓ = 8
Divide: ▓▓▓ = 4

x = 4

TYPE 3: VARIABLES ON BOTH SIDES

5x - 3 = 2x + 9
Step 1: Move variables to one side
5x - 2x = 9 + 3
Step 2: Simplify
3x = 12
Step 3: Divide
x = 4
Visual:
█████ - 3 = ██ + 915
5x
2x
Move 2x left, 3 right:
███ = 12
3x
Each █ = 4
x = 4

TYPE 4: WITH BRACKETS

3(x + 2) = 18
Step 1: Expand bracket
3x + 6 = 18
Step 2: Solve as usual
3x = 12
x = 4
Visual:
3 × [x + 2] = 18
Expand:
[x][x][x] + [2][2][2] = 18
3x
+
6
= 18

Diagram 13: Verification Process

ALWAYS CHECK YOUR ANSWER!
Example: If x = 4 for equation 5x - 7 = 13
SUBSTITUTE back into original equation:
Original: 5x - 7 = 13
↓
Put x = 4: 5(4) - 7 = 13
↓
Calculate: 20 - 7 = 13
↓16
Result:

13 = 13 ✓
Visual Check:

```
┌──────────────────┐
┌──────┐
 Left Side
 │Right │
 5(4) - 7 = 13
=?│ 13 │
└──────────────────┘
└──────┘
```

✓ EQUAL ✓
If they match → CORRECT!
If they don't → ERROR, try again!

Diagram 17: Common Shape Formulas

RECTANGLE

```
┌──────────────────┐
│
│
│ w (width)
│
│
└──────────────────┘
```

l (length)
Perimeter = 2l + 2w = 2(l + w)
Area = l × w
Example: l = 8cm, w = 5cm
P = 2(8 + 5) = 26 cm
A = 8 × 5 = 40 cm^2

SQUARE

```
┌───────────┐
│
│
│ s23
│
│
└───────────┘
```

s
? Perimeter = 4s

? Area = s^2

Example: s = 6cm

$P = 4 \times 6 = 24$ cm

$A = 6^2 = 36$ cm^2

TRIANGLE

```
/\
/ \
/
\ h (height)
/
\
/
\
/__________\
b (base)
```

? Perimeter = a + b + c

? Area = $\frac{1}{2} \times b \times h$

Example: b = 10cm, h = 6cm

$A = \frac{1}{2} \times 10 \times 6 = 30$ cm^2

CIRCLE

```
___
/
\
|
•
| r (radius)
|
| d (diameter)
\_____/
d = 2r24
```

? Circumference = $2\pi r = \pi d$

? Area = πr^2

Example: r = 7cm, $\pi = 22/7$

$C = 2 \times 22/7 \times 7 = 44$ cm

$A = 22/7 \times 7^2 = 154$ cm^2

TRAPEZIUM

```
a (top)
__________
/
\
/
\ h (height)
```

```
/
\
/__________________\
b (bottom)
? Area = ½(a + b) × h
Example: a = 5cm, b = 9cm, h = 4cm
A = ½(5 + 9) × 4 = 28 cm²
─────────────────────────────

PARALLELOGRAM

__________
/
/
/
/ h (height)
/_________/
b (base)
? Area = b × h
Example: b = 12cm, h = 5cm
A = 12 × 5 = 60 cm²
```
[25]

Worked Examples:

Example 1: Simplify: 5x + 3y - 2x + 7y

Diagram 18: Step-by-Step Visual Solution

```
ORIGINAL EXPRESSION:
┌──────────────────────────┐
  5x + 3y - 2x + 7y
│
└──────────────────────────┘
STEP 1: Color-code like terms
┌──────────────────────────┐
  5x + 3y - 2x + 7y
│
│ ? ? ? ?

  (red = x, blue = y)
│
└──────────────────────────┘
STEP 2: Rearrange (group like terms)
┌──────────────────────────┐
  (5x - 2x) + (3y + 7y)
│
│ ?
?
│
└──────────────────────────┘
STEP 3: Combine x terms
```

```
5x - 2x = ?
Visual:
█████ (5x)
- ██ (2x)
─────
███
(3x)
STEP 4: Combine y terms
3y + 7y = ?
Visual:
███
(3y)
+ ███████ (7y)
──────────
██████████ (10y)
```
26

```
STEP 5: Write final answer
┌──────────────────────────┐
│
ANSWER: 3x + 10y
│
└──────────────────────────┘
CHECK:
Original had 4 terms: ✓
Final has 2 terms: ✓
All x's and y's grouped: ✓
```

Example 2: Expand and simplify: 3(2x - 5) + 4(x + 3)

Diagram 19: Bracket Expansion Visual

```
ORIGINAL: 3(2x - 5) + 4(x + 3)
PART A: Expand 3(2x - 5)
┌─────────────────────┐
│ 3 × (2x - 5)
│
│
↓
↓
↓
│
3 × 2x = 6x
│
```

```
│
3 × (-5) = -15
│
└─────────────────────┘
Result: 6x - 15
PART B: Expand 4(x + 3)
┌─────────────────────┐
│ 4 × (x + 3)
│
│
│
│
│
│
↓
↓
↓
│
4 × x = 4x
│
4 × 3 = 12
│
└─────────────────────┘
Result: 4x + 12
COMBINE BOTH PARTS:
┌───────────────────────────┐27
│ 6x - 15 + 4x + 12
│
│ ?
?
│
?
?
│
└───────────────────────────┘
GROUP LIKE TERMS:
┌───────────────────────────┐
│ (6x + 4x) + (-15 + 12)
│
10x + (-3)
```

```
│
└───────────────────────────┘
```

FINAL ANSWER:

```
┌───────────────────────────┐
│
10x - 3
│
└───────────────────────────┘
```

Number line check:
-15
-3
0
+12

```
●─────────●───────┼─────────●
```

-3

Example 3: Solve for x: 5x - 7 = 3x + 9

Diagram 20: Equation Solving with Balance

ORIGINAL EQUATION:
5x - 7 = 3x + 9
VISUAL BALANCE:

```
┌─────────┐
┌─────────┐
│ 5x - 7 │ = │ 3x + 9 │
└────┬────┘
└────┴────┘
┴────┴─────────┴
══════════════
```

BALANCED
STEP 1: Move 3x to left side
(Subtract 3x from both sides)
5x - 7 = 3x + 9
-3x
-3x
──────────────
2x - 7 = 9
Visual:

```
┌─────────┐
┌────┐
│ 2x - 7 │ = │ 9 │
└────┬────┘
└──┬─┘
┴──┴──────┴
═══════════
```

STILL BALANCED
STEP 2: Move -7 to right side
(Add 7 to both sides)
2x - 7 = 9
+7

```
+7
──────────
2x = 16
Visual:
 ┌────┐
 │ 2x │  = │ 16 │
 └─┬──┘
 └─┴──┘
═╧════════╧═
STILL BALANCED
STEP 3: Divide both sides by 2
2x = 16
÷2
÷2
───────
x = 8
Visual:
 ┌───┐
 │ x │  = │ 8 │
 └─┬─┘
 └─┴─┘
═╧════════╧═
BALANCED!
VERIFICATION:
Substitute x = 8:29
Left side: 5(8) - 7 = 40 - 7 = 33
Right side: 3(8) + 9 = 24 + 9 = 33
33 = 33 ✓ CORRECT!
```

Example 4: Convert 0.65 to a percentage and a fraction.

Diagram 21: Decimal Conversion Process

```
GIVEN: 0.65
CONVERSION 1: Decimal → Percentage
┌─────────────────────────────┐
│ Step 1: Multiply by 100
│
│ 0.65 × 100 = 65
│
│
│
│ Step 2: Add % symbol
│
│ 65%
│
└─────────────────────────────┘
```

Visual (percentage bar):
0%
65%
100%

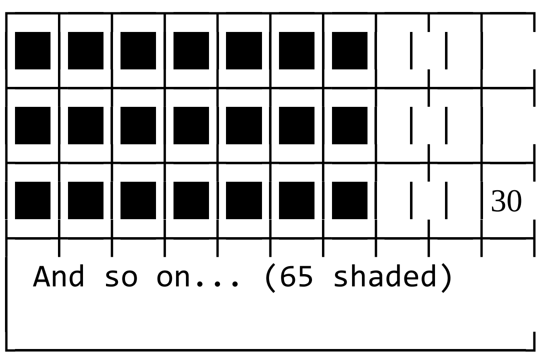
65 out of 100
CONVERSION 2: Decimal → Fraction

Step 1: Write as fraction
0.65 = 65/100

↑
↑

number decimal places

Visual (grid):
Grid showing 100 squares:



65 squares shaded out of 100 total

Step 2: Simplify fraction
Find GCF of 65 and 100

GCF = 5

65 ÷ 5 = 13

100 ÷ 5
20

FINAL ANSWERS:

Percentage: 65%

Fraction: 13/20

SUMMARY DIAGRAM:
0.65
/ \
/
\
/
\
65%
13/20

Example 5: Find the area of a rectangle with length 12 cm and width 8 cm.

Diagram 22: Rectangle Area Calculation

GIVEN RECTANGLE:
12 cm

8 |
| 8 cm
cm |

12 cm31
FORMULA:

Area = length × width

A = l × w

SUBSTITUTE VALUES:

A = 12 cm × 8 cm

A = 96 cm²

VISUAL GRID METHOD:
Break into 1cm × 1cm squares:

```
12 columns
┌──┬──┬──┬──┬──┬──┬──┬──┬──┬──┬──┬──┐ ← 8
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
r
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
o
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
w
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
s
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
└──┴──┴──┴──┴──┴──┴──┴──┴──┴──┴──┴──┘
```

Count squares:
• Each row: 12 squares
• Number of rows: 8
• Total: $12 \times 8 = 96$ squares
Each square = 1 cm^2
Total Area = 96 cm^2

WEEK 3: NUMBERS AND APPROXIMATION

Learning Objectives:

By the end of the lesson, students should be able to:
1. Express numbers in standard form
2. Round numbers to specified significant figures
3. Calculate percentage errors in measurements
4. Apply approximation in real-life situations

Entry Behaviour:

Students can perform basic arithmetic operations and understand place values in the decimal system.

Instructional Materials:

- Scientific calculator
- Charts showing powers of 10
- Number line
- Whiteboard and markers

Content:

A. Standard Form (Scientific Notation)

Diagram 30: Understanding Standard Form

STANDARD FORM: $A \times 10^n$
Where: $1 \leq A < 10$ and n is an integer
STRUCTURE:

```
┌──────────────────────────────────┐
│ A × 10^n
│
│ ↑
```

↑
│
│ │
└─ Power of 10
│
│ └─ Number between 1 and 10
│
└──────────────────────────────────┘

EXAMPLES:39

Large Number: 5,600,000

```
┌───────────────────────────────┐
│ 5,600,000 = 5.6 × 1,000,000 │
│
= 5.6 × 10⁶
│
│
│ Move decimal 6 places left: │
│ 5600000. → 5.6
│
│ ←←←←←←
│
└───────────────────────────────┘
```

Small Number: 0.00034

```
┌───────────────────────────────┐
│ 0.00034 = 3.4 × 0.0001
│
│
= 3.4 × 10⁻⁴
│
│
│ Move decimal 4 places right: │
│ 0.00034 → 3.4
│
→→→→
│
└───────────────────────────────┘
```

Diagram 31: Powers of 10 Reference

POSITIVE POWERS (Large Numbers)

10^6 = 1,000,000
(Million)
10^5 = 100,000
(Hundred thousand)

$10^4 = 10,000$
(Ten thousand)
$10^3 = 1,000$
(Thousand)
$10^2 = 100$
(Hundred)
$10^1 = 10$
(Ten)
$10^0 = 1$
(One)
NEGATIVE POWERS (Small Numbers)

$10^{-1} = 0.1$
(One tenth)
$10^{-2} = 0.01$
(One hundredth)
$10^{-3} = 0.001$
(One thousandth)
$10^{-4} = 0.0001$
(Ten thousandth)
$10^{-5} = 0.00001$
(Hundred thousandth)
$10^{-6} = 0.000001$
(Millionth)40
NUMBER LINE VIEW:
←─────────────────┼─────────────────→
Smaller
10^0=1
Larger
(negative)
(positive)
powers
powers

B. Significant Figures

Diagram 32: Counting Significant Figures

RULES FOR SIGNIFICANT FIGURES:
Rule 1: Non-zero digits are ALWAYS significant
234 → 3 s.f. (all count)
█ █ █
2 3 4
Rule 2: Zeros BETWEEN non-zeros are significant
2003 → 4 s.f. (zeros count)
█ ██ █
2 00 3
Rule 3: LEADING zeros are NOT significant
0.0025 → 2 s.f. (leading zeros don't count)

░░██
0025
Rule 4: TRAILING zeros in decimals ARE significant
2.500 → 4 s.f. (trailing zeros count)
█ ███
2 500
VISUAL EXAMPLES:
Number
| Significant Figures | Count

345
| █ █ █
| 3 s.f.
20.03
| █ ██ █
| 4 s.f.
0.0056
| ░░░█ █
| 2 s.f.
1.200
| █ ███
| 4 s.f.
500
| █░░ (ambiguous!)
| 1 or 3 s.f.41

Diagram 33: Rounding Process

```
ROUNDING TO 3 SIGNIFICANT FIGURES
Example: 0.035682 → 3 s.f.
STEP 1: Identify first 3 s.f.
0.035682
░░███↓
356 (8 is next digit)
STEP 2: Look at next digit
Next digit = 8
┌──────────────────┐
│ If ≥ 5: Round UP │
│ If < 5: Round DN │
└──────────────────┘
STEP 3: Since 8 ≥ 5, round up
356 → 357
STEP 4: Write final answer
0.0357
VISUAL GUIDE:
Number line showing 356 and 357:
356├────────┼────────┤357
0 1 2 3 4 5 6 7 8 9
```

↑
8 is here
(closer to 357)

C. Percentage Error

Diagram 34: Percentage Error Concept

```
FORMULA:
┌───────────────────────────────────────┐
│
│42
 % Error = |Approximate - Exact| ×100|
│
──────────────────
│
Exact
│
└───────────────────────────────────────┘
VISUAL REPRESENTATION:
Target vs Actual:
┌─────┐ ← Exact value
 ●
 ← Approximate value
└─────┘
↕
Error (difference)
Example: Measuring a desk
Measured: 1.23 m
Actual:
1.25 m
┌──────────────┐ Actual
├──────────────┤ Measured
└──────────────┘
Error = 0.02 m
CALCULATION STEPS:
┌─────────────────────────────┐
 Step 1: Find difference

 |1.23 - 1.25| = 0.02

├─────────────────────────────┤
 Step 2: Divide by exact

 0.02 ÷ 1.25 = 0.016
│
```

Step 3: Multiply by 100

0.016 × 100 = 1.6%

Worked Examples:

Example 1: Express 0.000456 in standard form

Diagram 35: Solution Steps[43]

Given: 0.000456
Step 1: Move decimal to get $1 \le A < 10$
0.000456 → 4.56
→→→→
(4 places right)
Step 2: Count moves
Moved right = negative power
4 places = 10^{-4}
Step 3: Write answer

$\boxed{4.56 \times 10^{-4}}$

Visual check:
$10^{-4} = 0.0001$
4.56 × 0.0001 = 0.000456 ✓

Example 2: Round 3.14159 to 4 s.f.

Diagram 36: Rounding Visual

Given: 3.14159
Identify 4 s.f.:
3.14159
[redacted] [redacted]↓
3 1415 (9 is next)
Decision point:
3.1415[9] ← Next digit is 9
9 ≥ 5 → ROUND UP
3.1415 + 0.0001 = 3.1416
Answer: 3.142
(rounded to 4 s.f.)[44]
Number line:
3.1415├──┼──┤3.1416
5
9
↑
↑
midpoint here

Example 3: Calculate percentage error when $\pi \approx 3.14$

Diagram 37: Error Calculation

```
Given:
Approximate: 3.14
Actual: 3.14159
Visual comparison:
Exact: ├─────────────────┤ 3.14159
Approx:  ├─────────────┤ 3.14
└─────────────┘
      ┬
Error: 0.00159
Calculation:
% Error = |3.14 - 3.14159| × 100
─────────────────
3.14159
= 0.00159 × 100
─────────────
3.14159
= 0.051%
Result: Error is about 0.05%
(Very small - good approximation!)
```

Evaluation Questions:

1. Express 0.00072 in standard form.
2. Round 0.0078234 to 3 significant figures.
3. A measurement is 48.5 kg; actual is 50 kg. Find % error.

Assignment:

1. Express in standard form: (a) 0.00072 (b) 345,00045
2. Round 0.0078234 to 3 significant figures.
3. Calculate $(4.5 \times 10^6) \div (1.5 \times 10^3)$ in standard form.
4. Find percentage error when $\pi \approx 3.14$ (actual: 3.14159).

WEEK 4: SEQUENCES AND SERIES

Learning Objectives:

By the end of the lesson, students should be able to:

1. Define sequences and series
2. Find the nth term of arithmetic progressions
3. Calculate the sum of arithmetic series
4. Solve real-life problems involving AP

Entry Behaviour:

Students can identify patterns in numbers and perform basic algebraic operations.

Instructional Materials:

- Number pattern charts
- Whiteboard and markers
- Calculator

Content:

A. Arithmetic Progression (AP)

Diagram 43: Understanding AP

```
ARITHMETIC PROGRESSION:
```

Sequence where difference between consecutive terms is constant

Example: 3, 7, 11, 15, 19, ...

Visual:

3 → 7 → 11 → 15 → 19

+4 +4

+4

+4

←

Common difference (d)

FORMULA FOR NTH TERM:

$T_n = a + (n-1)d$

51

Where:

a = first term

d = common difference

n = term number

VISUAL MODEL:

Term 1: █

Term 2: █ + d

Term 3: █ + d + d

Term 4: █ + d + d + d

Diagram 44: Sum of AP

SUM FORMULA:

$S_n = n/2[2a + (n-1)d]$

OR

$S_n = n/2(a + l)$

Where l = last term

```
└─────────────────────────────┘
VISUAL EXAMPLE:
Sum of 2, 5, 8, 11, 14
Pairing method:
2 + 14 = 16
5 + 11 = 16
8 stays in middle
Total = (16 × 2) + 8 = 40
OR using formula:
n=5, a=2, d=3
S₅ = 5/2[2(2) + 4(3)]
= 5/2[4 + 12]
= 5/2 × 16 = 40
```

Worked Examples:

Example 1: Find the 20th term of: 7, 11, 15, 19, ...

Diagram 45: Solution Steps

```
Given:
First term (a) = 7
Common difference (d) = 4
n = 20
Formula: Tₙ = a + (n-1)d
Substitute:
T₂₀ = 7 + (20-1)4
= 7 + 19(4)
= 7 + 76
= 83
Visual check:
Term 1: 7
Term 2: 11 (+4)
Term 3: 15 (+4)
...
Term 20: 83
Pattern:
├──┼──┼──┼──────────┼──┤
7 11 15 19 ... 79 83
└────────────────────┘
19 steps of +4
```

Evaluation Questions:

1. Find the common difference of AP: 12, 17, 22, 27, ...
2. Calculate the 25th term of AP: 4, 9, 14, 19, ...
3. Find the sum of the first 20 terms of AP: 2, 5, 8, 11, ...

Assignment:

1. Find the 30th term of AP: -5, -1, 3, 7, ...
2. The sum of first 8 terms is 156, first term is 6. Find d.

523. A theater has 20 seats in first row, 24 in second row, 28 in third. How many seats in 15 rows total?

WEEK 5: SEQUENCES AND SERIES II

Learning Objectives:

By the end of the lesson, students should be able to:
1. Define sequences and series
2. Find the nth term of geometric progressions
3. Calculate the sum of geometric series
4. Solve real-life problems involving GP

Entry Behaviour:

Students can identify patterns in numbers and perform basic algebraic operations.

Instructional Materials:

- Number pattern charts
- Whiteboard and markers
- Calculator

Content:

```
2. GEOMETRIC SEQUENCE (GP)
┌──────────────────────────────┐
│ Constant ratio between
│
│ consecutive terms
│
└──────────────────────────────┘
Example: 2, 6, 18, 54, 162, ...
×3 ×3 ×3 ×3
↑ ↑
↑
↑
Common ratio = 3
```

```
3. FIBONACCI SEQUENCE
┌──────────────────────────────┐ 86
│ Each term is sum of
│
│ previous two terms
│
└──────────────────────────────┘
Example: 1, 1, 2, 3, 5, 8, 13, ...
↓ ↓ ↓ ↓ ↓ ↓
1+1=2
1+2=3
2+3=5
3+5=8
5+8=13
```

```
4. OTHER SEQUENCES
Square numbers:
1, 4, 9, 16, 25, ...
```

1^2 2^2 3^2 4^2 5^2
Cube numbers:
1, 8, 27, 64, 125, ...
1^3 2^3 3^3 4^3
5^3
Triangular numbers:
1, 3, 6, 10, 15, ...

C. Arithmetic Progression (AP) - Detailed Study

Diagram 66: AP Components

ARITHMETIC PROGRESSION:
Sequence where each term differs from
the next by a constant amount[87]
GENERAL FORM:
a, a+d, a+2d, a+3d, a+4d, ...
Where:
• a = first term
• d = common difference
• n = position/term number
FINDING COMMON DIFFERENCE (d):
Formula: $d = T_2 - T_1 = T_3 - T_2 = T_n - T_{n-1}$
Example: 5, 8, 11, 14, 17, ...
d = 8 - 5 = 3
d = 11 - 8 = 3
d = 14 - 11 = 3
+3
Constant difference = 3
TYPES OF AP:
1. INCREASING AP (d > 0)
2, 5, 8, 11, 14, ...
↗ ↗ ↗ ↗ (going up)
2. DECREASING AP (d < 0)
20, 15, 10, 5, 0, ...
↘ ↘ ↘ ↘ (going down)
3. CONSTANT AP (d = 0)
7, 7, 7, 7, 7, ...
→ → → → (staying same)[88]
NUMBER LINE VIEW:
Increasing AP (d=3):
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
0 2 5 8 11 14 17 20 23 26 30
←3→←3→←3→
Decreasing AP (d=-4):
├──┼──┼──┼──┼──┼──┼──┼──┼──┼──┤
0 4 8 12 16 20 24 28 32 36 40
→←4←←4←←4←

20 16 12 8 4

Diagram 67: nth Term Formula Derivation

FINDING THE NTH TERM:
Pattern analysis:
Term 1: a = a + 0d
Term 2: a + d = a + 1d
Term 3: a + 2d = a + 2d
Term 4: a + 3d = a + 3d
Term 5: a + 4d = a + 4d
...
Term n: a + (n-1)d
FORMULA:

$\boxed{T_n = a + (n-1)d}$

Where:

T_n = nth term

a = first term

n = position

d = common difference

VISUAL DERIVATION:
Example: 3, 7, 11, 15, ...
a = 3, d = 4Term 1: 3 = 3 + (1-1)×4 = 3 + 0
└─ 0 steps of 4
Term 2: 7 = 3 + (2-1)×4 = 3 + 4
└─ 1 step of 4
Term 3: 11 = 3 + (3-1)×4 = 3 + 8
└─ 2 steps of 4
Term 4: 15 = 3 + (4-1)×4 = 3 + 12
└─ 3 steps of 4
Term n: ? = 3 + (n-1)×4
└─ (n-1) steps of 4

STEP-BY-STEP:
Start at 'a': ●
After 1 step: ●───→ a + d
After 2 steps: ●───→───→ a + 2d
After 3 steps: ●───→───→───→ a + 3d
...
After (n-1) steps: a + (n-1)d

Diagram 68: Finding Unknown Terms

TYPES OF PROBLEMS:
PROBLEM TYPE 1: Find specific term
Given: a, d, n
Find: T_n
Example: a=5, d=3, find T_{10}
T_n = a + (n-1)d
T_{10} = 5 + (10-1)×3
T_{10} = 5 + 27 = 32
8990

PROBLEM TYPE 2: Find first term
Given: T_n , d, n
Find: a
Example: T_8=35, d=4, find a
35 = a + (8-1)×4
35 = a + 28
a = 7

PROBLEM TYPE 3: Find common difference
Given: a, T_n , n
Find: d
Example: a=3, T_{12}=47, find d
47 = 3 + (12-1)d
47 = 3 + 11d
44 = 11d
d = 4

PROBLEM TYPE 4: Find position
Given: a, d, T_n
Find: n
Example: a=2, d=5, T_n =52, find n
52 = 2 + (n-1)×5
52 = 2 + 5n - 5
52 = 5n - 3
55 = 5n
n = 11
FORMULA TRIANGLE:

T_n
/ \
/
\91
/ ——— \
a (n-1)d
To find any part, cover it and see what remains:

- $T_n = a + (n-1)d$
- $a = T_n - (n-1)d$
- $d = (T_n - a)/(n-1)$

D. Sum of Arithmetic Series

Diagram 69: Understanding Series Sum

SERIES vs SEQUENCE:
SEQUENCE: List of terms
3, 7, 11, 15, 19, ...
SERIES: Sum of terms
3 + 7 + 11 + 15 + 19 + ...
SUM OF FIRST n TERMS (S_n):
METHOD 1: PAIRING METHOD
Example: Sum of 2, 5, 8, 11, 14
Write forward: 2 + 5 + 8 + 11 + 14
Write backward: 14 + 11 + 8 + 5 + 2
Add pairs:
16 + 16 + 16 + 16 + 16
5 pairs of 16 = 5 × 16 = 80
But we added twice, so:
Sum = 80 ÷ 2 = 40
Visual:
2————————14 = 16
5———————11 = 16
8——————8
= 16
FORMULA 1:
92

$S_n = n/2 \times [2a + (n-1)d]$

Where:

S_n = sum of n terms

n = number of terms

a = first term

d = common difference

FORMULA 2 (Alternative):

$S_n = n/2 \times (a + l)$

Where:

l = last term (nth term)

WHEN TO USE EACH FORMULA:

Formula 1: When you know a, d, n

Formula 2: When you know first and last terms

VISUAL MODEL:

Sum of 1, 3, 5, 7, 9 (5 terms)

●
● ● ●
● ● ● ● ●
● ● ● ● ● ● ●
● ● ● ● ● ● ● ● ●

This forms a square pattern:

$5 \times 5 = 25$ (answer)

Or using formula:

$S_n = n/2 \times (\text{first} + \text{last})$

$S_5 = 5/2 \times (1 + 9)$

$= 5/2 \times 10 = 25$

Diagram 70: Sum Formula Derivation

DERIVING THE SUM FORMULA:

Let S = sum of AP: a, a+d, a+2d, ..., a+(n-1)d

Write forward:

$S = a + (a+d) + (a+2d) + ... + [a+(n-1)d]$

Write backward:

$S = [a+(n-1)d] + [a+(n-2)d] + ... + a$

Add both lines:

$2S = \underbrace{[2a+(n-1)d] + [2a+(n-1)d] + ...}_{\text{n times}} + [2a+(n-1)d]$

$2S = n \times [2a + (n-1)d]$

$S = n/2 \times [2a + (n-1)d]$

ALTERNATIVE FORM:
Since last term $l = a + (n-1)d$
We can write:
$2a + (n-1)d = a + [a + (n-1)d]$
$= a + l$
Therefore:
$S = n/2 \times (a + l)$
MEMORY AID:
S_n = (Number of terms/2) × (First + Last)
= Average × Number of terms
Visual:
n terms

S | ← Total sum

↑
$n/2 \times (a + l)$
Average height94

E. Applications and Problem-Solving

Diagram 71: Real-Life Applications

APPLICATION 1: SAVINGS PATTERN

A person saves ₦500 in month 1, ₦650 in month 2, ₦800 in

month 3, and so on.

This forms an AP!

Month: 1
Save: ₦500 ₦650 ₦800 ₦950 ₦1100
+150 +150 +150 +150
$a = 500$, $d = 150$
Question: How much in month 12?
$T_{12} = 500 + (12-1)\times 150$
$= 500 + 1650$
= ₦2,150
Question: Total saved in 12 months?
$S_{12} = 12/2 \times [2(500) + 11(150)]$
$= 6 \times [1000 + 1650]$
$= 6 \times 2650$

= ₦15,900

APPLICATION 2: SEATING ARRANGEMENT

A theater has:

Row 1: 20 seats

Row 2: 24 seats

Row 3: 28 seats

Pattern continues...

Visual:95

Row 1: ●●●●●●●●●●●●●●●●●●●● (20)

Row 2: ●●●●●●●●●●●●●●●●●●●●●●●● (24)

Row 3: ●●●●●●●●●●●●●●●●●●●●●●●●●●●● (28)

$a = 20$, $d = 4$

Question: Seats in row 15?

$T_{15} = 20 + (15-1)\times4$

$= 20 + 56$

$= 76$ seats

Question: Total seats in 15 rows?

$S_{15} = 15/2 \times (20 + 76)$

$= 15/2 \times 96$

$= 720$ seats

APPLICATION 3: STACKING OBJECTS

Cans stacked in rows:

Top row: 3 cans

Next row: 5 cans

Next row: 7 cans

Pattern continues...

$a = 3$, $d = 2$

Question: Cans in 10th row?

$T_{10} = 3 + (10-1)\times2$

$= 3 + 18$

= 21 cans96
Question: Total cans in 10 rows?
S_{10} = 10/2 × [2(3) + 9(2)]
= 5 × [6 + 18]
= 5 × 24
= 120 cans

APPLICATION 4: DISTANCE COVERED

A car travels:
Hour 1: 50 km
Hour 2: 55 km
Hour 3: 60 km
Speed increases constantly

a = 50, d = 5
Question: Distance in 8th hour?
T_8 = 50 + (8-1)×5
= 50 + 35
= 85 km
Question: Total distance in 8 hours?
S_8 = 8/2 × [2(50) + 7(5)]
= 4 × [100 + 35]
= 4 × 135
= 540 km

Worked Examples with Complete Solutions:

Example 1: Find the 20th term of the AP: 7, 11, 15, 19, ...

Diagram 72: Solution Process

Given AP: 7, 11, 15, 19, ...
STEP 1: Identify components
First term (a) = 797
Common difference (d) = 11 - 7 = 4
Check: 15 - 11 = 4 ✓
19 - 15 = 4 ✓
STEP 2: Identify what to find
n = 20 (we want 20th term)
Find: T_{20}
STEP 3: Choose formula
T_n = a + (n-1)d
STEP 4: Substitute values

$T_{20} = 7 + (20-1) \times 4$
$= 7 + 19 \times 4$
$= 7 + 76$
$= 83$
STEP 5: Verify answer is reasonable
Pattern: 7, 11, 15, 19, ...
Each term increases by 4
After 19 jumps of 4: 7 + 76 = 83 ✓
VISUAL VERIFICATION:
Term 1: 7 = 7 + 0×4
Term 2: 11 = 7 + 1×4
Term 3: 15 = 7 + 2×4
Term 4: 19 = 7 + 3×4
...
Term 20: 83 = 7 + 19×4
NUMBER LINE:

```
├──┼──┼──┼──────────┼──┤
7 11 15 19
...
83
└─────────────────────┘
```

19 steps of 4
ANSWER: The 20th term is 83
ALTERNATIVE METHOD (Pattern):From term 1 to term 20: 19 steps
Each step = 4
Total increase = 19 × 4 = 76
Starting point = 7
Final value = 7 + 76 = 83 ✓

Example 2: The 5th term of an AP is 23 and the 12th term is 58. Find the first term and common difference.

Diagram 73: Two-Equation Solution

Given:
$T_5 = 23$
$T_{12} = 58$
STEP 1: Write equations using formula
$T_n = a + (n-1)d$
For 5th term:
$T_5 = a + (5-1)d$
23 = a + 4d ... equation (1)
For 12th term:
$T_{12} = a + (12-1)d$
58 = a + 11d ... equation (2)
STEP 2: Solve simultaneously
Subtract equation (1) from equation (2):
58 = a + 11d

-(23 = a + 4d)

---

35 = 0 + 7d
35 = 7d
d = 5
STEP 3: Find first term
Substitute d = 5 into equation (1):
23 = a + 4(5)
23 = a + 20
a = 3
STEP 4: Verify
Check with T_5:
$T_5 = 3 + 4(5) = 3 + 20 = 23$ ✓
Check with T_{12}:
$T_{12} = 3 + 11(5) = 3 + 55 = 58$ ✓
VISUAL REPRESENTATION:

```
Position: 1 2 3 4 5 6 7 8 9 10 11 12
┌─┬─┬─┬─┬──┬──┬──┬──┬──┬──┬──┬──┐
Value:
│3│8│13│18│23│28│33│38│43│48│53│58│
└─┴─┴──┴──┴──┴──┴──┴──┴──┴──┴──┘
+5 +5 +5 +5 +5 +5 +5 +5 +5 +5 +5
```

ANSWER:
First term (a) = 3
Common difference (d) = 5
The sequence is: 3, 8, 13, 18, 23, 28, ...
Example 3: Find the sum of the first 15 terms of the AP: 3, 7, 11, 15, ...
Diagram 74: Sum Calculation
Given AP: 3, 7, 11, 15, ...
STEP 1: Identify components
First term (a) = 3
Common difference (d) = 7 - 3 = 4Number of terms (n) = 15
STEP 2: Choose formula
Since we know a, d, and n:
$S_n = n/2 \times [2a + (n-1)d]$
STEP 3: Substitute values
$S_{15} = 15/2 \times [2(3) + (15-1) \times 4]$
$= 15/2 \times [6 + 14 \times 4]$
$= 15/2 \times [6 + 56]$
$= 15/2 \times 62$
$= 15 \times 31$
$= 465$
ALTERNATIVE METHOD (Using Formula 2):
First find last term:
$T_{15} = 3 + (15-1) \times 4$

= 3 + 56
= 59
Then use:
S_{15} = n/2 × (first + last)
= 15/2 × (3 + 59)
= 15/2 × 62
= 465 ✓
Total = 465 blocks
PAIRING METHOD VERIFICATION:
Pair first and last: 3 + 59 = 62
Pair second and 14th: 7 + 55 = 62
Pair third and 13th: 11 + 51 = 62
7.5 pairs × 62 = 465 ✓
(or 15 terms ÷ 2 × 62)
ANSWER: Sum of first 15 terms = 465
BREAKDOWN:

```
┌──────────────────────────────┐
 Terms to add:

 3+7+11+15+19+23+27+31+35+

 39+43+47+51+55+59



 Organized sum:

 (3+59) + (7+55) + (11+51) +...


62 + 62 + 62 + ...

 = 7.5 × 62 = 465

└──────────────────────────────┘
```

Example 4: How many terms of the AP 5, 8, 11, 14, ... must be taken so that their sum is 345?
Diagram 75: Finding Number of Terms
Given:
AP: 5, 8, 11, 14, ...
Sum (S_n) = 345
Find: n
STEP 1: Identify known values
First term (a) = 5
Common difference (d) = 8 - 5 = 3
Sum (S_n) = 345

STEP 2: Use sum formula
$S_n = n/2 \times [2a + (n-1)d]$
$345 = n/2 \times [2(5) + (n-1)\times 3]$
STEP 3: Simplify
$345 = n/2 \times [10 + 3n - 3]345 = n/2 \times [7 + 3n]$
$690 = n(7 + 3n)$
$690 = 7n + 3n^2$
STEP 4: Rearrange to quadratic
$3n^2 + 7n - 690 = 0$
STEP 5: Solve quadratic equation
Using factoring or formula:
Method 1 - Factoring:
$3n^2 + 7n - 690 = 0$
$(3n + 46)(n - 15) = 0$
$n = -46/3$ or $n = 15$
Since n must be positive:
$n = 15$
Method 2 - Quadratic formula:
$n = [-7 \pm \sqrt{(49 + 8280)}] / 6$
$n = [-7 \pm \sqrt{8329}] / 6$
$n = [-7 \pm 91.26] / 6$
$n = 84.26/6$ or $-98.26/6$
$n \approx 15$ (taking positive value)
ANSWER: 15 terms
VISUAL CHECK:
Term values: 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47
Count: 15 terms

Example 5: A man saves ₦500 in the first month, ₦650 in the second month, ₦800 in the third month, and so on. How much will he save in the 12th month, and what is his total savings after 12 months?

Diagram 76: Real-World Application

Given:
Month 1: ₦500
Month 2: ₦650
102103
Month 3: ₦800
Pattern continues...
STEP 1: Identify as AP
First term (a) = 500
Common difference (d) = 650 - 500 = 150
Check: 800 - 650 = 150 ✓
STEP 2: Find amount in 12th month
Use: $T_n = a + (n-1)d$
$T_{12} = 500 + (12-1) \times 150$
$= 500 + 11 \times 150$

= 500 + 1650
= ₦2,150
STEP 3: Find total savings after 12 months
Use: $S_n = n/2[2a + (n-1)d]$
$S_{12} = 12/2[2(500) + 11×150]$
= 6[1000 + 1650]
= 6 × 2650
= ₦15,900
│
ANSWER:
• Amount in 12th month: ₦2,150
• Total savings: ₦15,900
PRACTICAL NOTE:
This savings pattern increases by ₦150
each month. Good for building
emergency fund or major purchase!

Class Activity:

Activity 1: Sequence Detective

Diagram 77: Pattern Recognition

Identify the pattern and find the next 3 terms:
1. 4, 9, 14, 19, 24, ?, ?, ?
Pattern: AP with d = 5
Answer: 29, 34, 39
2. 100, 95, 90, 85, ?, ?, ?
Pattern: AP with d = -5
Answer: 80, 75, 70
3. 2, 6, 10, 14, ?, ?, ?
Pattern: AP with d = 4
Answer: 18, 22, 26
4. 50, 47, 44, 41, ?, ?, ?
Pattern: AP with d = -3
Answer: 38, 35, 32

Activity 2: AP Builder Create your own AP with:
• First term = your age
• Common difference = 3
• Write first 10 terms
• Find sum of first 10 terms

Activity 3: Real-World Scenarios

In groups, create word problems using AP for:
• Savings plan
• Ticket sales
• Construction project
• Sports training program

Evaluation Questions:

1. Find the common difference of the AP: 12, 17, 22, 27, ...

2. Calculate the 25th term of the AP: 4, 9, 14, 19, ...
3. The 3rd term of an AP is 18 and the 7th term is 34. Find the first term.
4. Find the sum of the first 20 terms of the AP: 2, 5, 8, 11, ...
5. How many terms of the AP 8, 12, 16, 20, ... will give a sum of 240?

Assignment:

1. Write the first five terms of an AP whose first term is 7 and common difference is -3.
2. Find the 30th term of the AP: -5, -1, 3, 7, ...
3. The sum of the first 8 terms of an AP is 156, and the first term is 6. Calculate the common difference.
4. An AP has first term 10 and last term 82. If the sum of all terms is 460, find the number of terms.
5. A theater has 20 seats in the first row, 24 seats in the second row, 28 seats in the third row, and so on. If there
are 15 rows, how many seats are there in total?

105106
5354

WEEK 6: QUADRATIC EQUATIONS I

Learning Objectives:

By the end of the lesson, students should be able to:

1. Define quadratic equations and identify their components
2. Form quadratic equations from given roots
3. Find sum and product of roots
4. Solve quadratic equations by factorization
5. Solve quadratic equations by completing the square method
6. Solve quadratic equations using the quadratic formula
7. Apply quadratic equations to real-life problems

Entry Behaviour:

Students can expand brackets, factorize algebraic expressions, and solve linear equations.

Instructional Materials:

- Whiteboard and markers
- Algebra tiles or manipulatives
- Graph paper
- Calculator
- Formula charts
- Worked examples sheets

Content:

A. Introduction to Quadratic Equations

Definition: A quadratic equation is an equation of the form $ax^2 + bx + c = 0$, where a, b, and c are constants and $a \neq 0$.

Diagram 78: Anatomy of a Quadratic Equation

```
STANDARD FORM:
ax² + bx + c = 0
Where:
┌──────────────────────────────────┐
│ a = coefficient of x²
```

```
│
│
(quadratic coefficient)
│107
│
Must NOT be zero (a ≠ 0)
│
│
│
│ b = coefficient of x
│
│
(linear coefficient)
│
│
│
│ c = constant term
│
│
(independent of x)
│
└──────────────────────────────────┘
VISUAL BREAKDOWN:
2x² + 5x - 3 = 0
│ │
│
│
│
│ │
│
│
└─ Equals zero
│ │
│
└───── Constant term (c = -3)
│ │
└───────── Linear term (b = 5)
│ └───────────── Coefficient (a = 2)
└─────────────── Variable squared
DEGREE OF EQUATION:
Highest power of variable = 2
Therefore: QUADRATIC (from Latin "quadratus" = square)
COMPARISON WITH OTHER EQUATIONS:
LINEAR (Degree 1):
ax + b = 0
Example: 2x + 3 = 0
```

QUADRATIC (Degree 2):
$ax^2 + bx + c = 0$
Example: $2x^2 + 3x + 1 = 0$
CUBIC (Degree 3):
$ax^3 + bx^2 + cx + d = 0$
Example: $2x^3 + 3x^2 + x + 1 = 0$
WHY QUADRATIC IS IMPORTANT:
• Models many real-world situations
• Area problems
• Projectile motion108
• Business profit calculations
• Optimization problems

Diagram 79: Identifying Coefficients

EXAMPLES OF IDENTIFYING a, b, c:
1. $x^2 + 5x + 6 = 0$
↓
↓
↓
a=1, b=5, c=6
2. $2x^2 - 3x + 1 = 0$
↓
↓
↓
a=2, b=-3, c=1
3. $x^2 - 4 = 0$
↓
↓
a=1, b=0, c=-4
(No x term means b=0)
4. $3x^2 + 7x = 0$
↓
↓
↓
a=3, b=7, c=0
(No constant means c=0)
5. $-x^2 + 2x - 5 = 0$
↓
↓
↓
a=-1, b=2, c=-5
(Negative x^2 means a=-1)
REARRANGING TO STANDARD FORM:
Before: $2x^2 = 3x - 5$
Step 1: Move all terms to left
$2x^2 - 3x + 5 = 0$
Result: a=2, b=-3, c=5

Before: $x(x + 4) = 12$
Step 1: Expand
$x^2 + 4x = 12$
Step 2: Rearrange$x^2 + 4x - 12 = 0$
Result: a=1, b=4, c=-12
Before: $(x - 3)(x + 2) = 0$
Step 1: Already in factored form
(can expand if needed)
$x^2 - x - 6 = 0$
Result: a=1, b=-1, c=-6
SPECIAL CASES:
Pure Quadratic (b=0):
$ax^2 + c = 0$
Example: $x^2 - 9 = 0$
Incomplete Quadratic (c=0):
$ax^2 + bx = 0$
Example: $x^2 + 5x = 0$
Complete Quadratic:
$ax^2 + bx + c = 0$ (all present)
Example: $x^2 + 5x + 6 = 0$

B. Roots of Quadratic Equations

Definition: The roots (or solutions) of a quadratic equation are the values of x that satisfy the equation.

Diagram 80: Understanding Roots

WHAT ARE ROOTS?
If $ax^2 + bx + c = 0$
And $x = \alpha$ and $x = \beta$ satisfy the equation
Then α and β are the ROOTS
VISUAL REPRESENTATION:
Equation: $x^2 - 5x + 6 = 0$
Factored: $(x - 2)(x - 3) = 0$
Solution:
$x - 2 = 0$ OR $x - 3 = 0$
$x = 2$
OR $x = 3$
Roots are: $\alpha = 2$ and $\beta = 3$
Roots (x-intercepts)
TYPES OF ROOTS:
1. TWO DISTINCT REAL ROOTS
Example: $x^2 - 5x + 6 = 0$
Roots: $x = 2$, $x = 3$
Two crossing points
2. TWO EQUAL ROOTS (Repeated Root)
Example: $x^2 - 4x + 4 = 0$
Roots: $x = 2$, $x = 2$

3. NO REAL ROOTS (Complex Roots)
Example: $x^2 + 2x + 5 = 0$
No real solutions
Graph:
Never crosses x-axis
TERMINOLOGY:
• Roots = Solutions = Zeros
• Finding roots = Solving equation
• x-intercepts = Roots on graph

Diagram 81: Sum and Product of Roots

RELATIONSHIP BETWEEN ROOTS AND COEFFICIENTS
For equation: $ax^2 + bx + c = 0$
With roots α and β
SUM OF ROOTS:

$$\boxed{\alpha + \beta = -b/a}$$

PRODUCT OF ROOTS:

$$\boxed{\alpha \times \beta = c/a}$$

VISUAL PROOF:
If $(x - \alpha)(x - \beta) = 0$Expand: $x^2 - (\alpha+\beta)x + \alpha\beta = 0$
Compare with: $x^2 + (b/a)x + (c/a) = 0$
(dividing standard form by a)
Matching coefficients:
$-(\alpha+\beta) = b/a \rightarrow \alpha+\beta = -b/a$
$\alpha\beta = c/a$
EXAMPLES:
1. $x^2 - 5x + 6 = 0$
a=1, b=-5, c=6
Sum: $\alpha+\beta = -(-5)/1 = 5$
Product: $\alpha\beta = 6/1 = 6$
Roots are 2 and 3:
Check: 2+3 = 5 ✓
2×3 = 6 ✓
2. $2x^2 + 7x - 4 = 0$
a=2, b=7, c=-4
Sum: $\alpha+\beta = -7/2 = -3.5$
Product: $\alpha\beta = -4/2 = -2$
3. $x^2 - 4x + 4 = 0$
a=1, b=-4, c=4
Sum: $\alpha+\beta = -(-4)/1 = 4$

Product: $\alpha\beta = 4/1 = 4$
Roots are 2 and 2:
Check: 2+2 = 4 ✓
2×2 = 4 ✓
MEMORY TRIANGLE:
EQUATION
$ax^2+bx+c=0$
α and β

C. Forming Quadratic Equations from Roots

Diagram 82: Building Equations from Roots

METHOD 1: USING FACTORS
If roots are α and β:
$(x - \alpha)(x - \beta) = 0$
EXAMPLE 1: Roots are 3 and 5
$(x - 3)(x - 5) = 0$
Expand:
$x^2 - 5x - 3x + 15 = 0$
$x^2 - 8x + 15 = 0$
EXAMPLE 2: Roots are -2 and 4
$(x - (-2))(x - 4) = 0$
$(x + 2)(x - 4) = 0$
Expand:
$x^2 - 4x + 2x - 8 = 0$
$x^2 - 2x - 8 = 0$

METHOD 2: USING SUM AND PRODUCT
If Sum = S and Product = P:
$x^2 - Sx + P = 0$
EXAMPLE 3: Sum = 7, Product = 12
$x^2 - 7x + 12 = 0$114
EXAMPLE 4: Sum = -3, Product = -10
$x^2 - (-3)x + (-10) = 0$
$x^2 + 3x - 10 = 0$

VISUAL PROCESS:
Given roots: 2 and 5
Step 1: Write factors
$(x - 2)(x - 5)$
Step 2: Expand using FOIL
F: $x \times x = x^2$
O: $x \times -5 = -5x$
I: $-2 \times x = -2x$
L: $-2 \times -5 = 10$
Step 3: Combine
$x^2 - 5x - 2x + 10 = 0$

$x^2 - 7x + 10 = 0$
EXPANSION BOX:
x
-5

```
 ┌────┬────┐
x │ x² │-5x │
 ├────┼────┤
-2 │-2x │ 10 │
 └────┴────┘
```

SPECIAL CASES:
1. Equal roots (both are k):
$(x - k)^2 = 0$
$x^2 - 2kx + k^2 = 0$Example: Both roots are 3
$(x - 3)^2 = 0$
$x^2 - 6x + 9 = 0$
2. Roots are opposites (k and -k):
$x^2 - k^2 = 0$
Example: Roots are 4 and -4
$x^2 - 16 = 0$
3. Fractional roots:
Roots: 1/2 and 3
$(x - 1/2)(x - 3) = 0$
Multiply through by 2:
$(2x - 1)(x - 3) = 0$
$2x^2 - 6x - x + 3 = 0$
$2x^2 - 7x + 3 = 0$

VERIFICATION:
After forming equation, verify by:
1. Calculating sum: -b/a
2. Calculating product: c/a
3. Should match given roots

D. Solving Quadratic Equations by Factorization

Diagram 83: Factorization Method

PRINCIPLE: Zero Product Property
If $A \times B = 0$, then $A = 0$ or $B = 0$
STEPS:
1. Write equation in standard form
2. Factorize left side
3. Set each factor to zero
4. Solve for x

115116

TYPE 1: Simple Trinomial (a = 1)
Pattern: $x^2 + bx + c = 0$

EXAMPLE: $x^2 + 5x + 6 = 0$

Step 1: Find two numbers that:

- Multiply to give c (6)
- Add to give b (5)

Numbers: 2 and 3

$(2 \times 3 = 6, 2 + 3 = 5)$ ✓

Step 2: Write factors

$(x + 2)(x + 3) = 0$

Step 3: Solve

$x + 2 = 0$ OR $x + 3 = 0$

$x = -2$

OR $x = -3$

FACTOR FINDER CHART:

For $x^2 + 5x + 6$:

Factors of 6 | Sum

---

1 × 6
| 7
2 × 3
| 5 ✓
-1 × -6
| -7
-2 × -3
| -5

Visual:

```
6
/ \
2
3
└───┘
  ┬
5 (sum)117
```

TYPE 2: Trinomial with $a \neq 1$

Pattern: $ax^2 + bx + c = 0$

EXAMPLE: $2x^2 + 7x + 3 = 0$

Method: Split the middle term

Step 1: Find $a \times c$

$2 \times 3 = 6$

Step 2: Find factors of 6 that add to 7

Factors: 6 and 1

$(6 + 1 = 7)$ ✓

Step 3: Rewrite middle term

$2x^2 + 6x + 1x + 3 = 0$

Step 4: Factor by grouping

$2x(x + 3) + 1(x + 3) = 0$

$(2x + 1)(x + 3) = 0$
Step 5: Solve
$2x + 1 = 0$ OR $x + 3 = 0$
$x = -1/2$
OR $x = -3$
GROUPING VISUAL:

```
2x² + 6x + 1x + 3
└──┬──┘  └──┬──┘
│
│
│
2x(x+3) 1(x+3)
└───┬───┘
(2x+1)(x+3)
```

TYPE 3: Difference of Squares
Pattern: $x^2 - k^2 = 0118$
EXAMPLE: $x^2 - 25 = 0$
Step 1: Recognize pattern
$x^2 - 5^2 = 0$
Step 2: Factor
$(x + 5)(x - 5) = 0$
Step 3: Solve
$x + 5 = 0$ OR $x - 5 = 0$
$x = -5$
OR $x = 5$
Visual:
$x^2 - 25$
↓
↓
$(x)^2$ $(5)^2$
↓
↓
$(x+5)(x-5)$

TYPE 4: Perfect Square Trinomial
Pattern: $x^2 \pm 2kx + k^2 = 0$
EXAMPLE: $x^2 + 6x + 9 = 0$
Step 1: Recognize pattern
$x^2 + 2(3)x + 3^2 = 0$
Step 2: Factor
$(x + 3)^2 = 0$
Step 3: Solve
$x + 3 = 0$
$x = -3$ (repeated root)
Visual:
$x^2 + 6x + 9$

(x+3)²

TYPE 5: Common Factor First
EXAMPLE: $2x^2 + 8x = 0$
Step 1: Factor out GCF
$2x(x + 4) = 0$
Step 2: Solve
$2x = 0$ OR $x + 4 = 0$
$x = 0$
OR $x = -4$
Warning: Don't divide by x!
(Would lose $x = 0$ solution)
FACTORIZATION DECISION TREE:
Quadratic

E. Completing the Square Method

Diagram 84: Completing the Square Concept

CONCEPT: Convert to perfect square form
$(x + p)^2 = q$
Then solve by taking square root[120]
VISUAL MODEL:
$x^2 + 6x + ?$
Think of this as area:

```
x
6
┌──┬─────┐
x │x²│ 6x │
└──┴─────┘
```

To complete square, split 6x:

```
x
3
3
┌──┬───┬───┐
x │x²│3x │3x │
├──┼───┼───┤
3 │3x│ 9 │
│← Need to add 9
└──┴───┴───┘
```

$x^2 + 6x + 9 = (x + 3)^2$

FORMULA METHOD:
For $x^2 + bx$:
Add $(b/2)^2$ to complete square
STEPS FOR $ax^2 + bx + c = 0$:
Step 1: Make coefficient of x^2 equal to 1
(divide by a if $a \neq 1$)
Step 2: Move constant to right side

Step 3: Add $(b/2)^2$ to both sides
Step 4: Factor left side as perfect square
Step 5: Take square root of both sides
Step 6: Solve for x

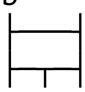
EXAMPLE 1: $x^2 + 6x + 5 = 0$
Step 1: Coefficient of x^2 is already 1
Step 2: Move constant
$x^2 + 6x = -5$
Step 3: Complete square
Half of 6 = 3
$3^2 = 9$
$x^2 + 6x + 9 = -5 + 9$
$x^2 + 6x + 9 = 4$
Step 4: Factor left side
$(x + 3)^2 = 4$
Step 5: Take square root
$x + 3 = \pm 2$
Step 6: Solve
$x + 3 = 2$ OR $x + 3 = -2$
$x = -1$
OR $x = -5$
VISUAL STEPS:
$x^2 + 6x + 5 = 0$
↓
$x^2 + 6x = -5$
↓ (add 9)
$x^2 + 6x + 9 = 4$
↓
$(x + 3)^2 = 4$
↓ (√)
$x + 3 = \pm 2$
↓
$x = -1$ or -5

EXAMPLE 2: $2x^2 + 8x - 10 = 0$
Step 1: Divide by 2
$x^2 + 4x - 5 = 0$
Step 2: Move constant
$x^2 + 4x = 5$
Step 3: Complete square
$(4/2)^2 = 4$
$x^2 + 4x + 4 = 5 + 4$
$x^2 + 4x + 4 = 9$
Step 4: Factor
$(x + 2)^2 = 9$
Step 5: Square root

$x + 2 = \pm 3$
Step 6: Solve
$x = -2 + 3 = 1$
OR
$x = -2 - 3 = -5$
COMPLETING SQUARE FORMULA:

$$\boxed{x^2 + bx + (b/2)^2 = (x+b/2)^2}$$

NUMBER LINE:
Finding (b/2):
b



b/2 b/2

EXAMPLE 3: $x^2 - 5x + 3 = 0$
122123
Step 1: Move constant
$x^2 - 5x = -3$
Step 2: Complete square
$(-5/2)^2 = 25/4$
$x^2 - 5x + 25/4 = -3 + 25/4$
$x^2 - 5x + 25/4 = (-12+25)/4$
$x^2 - 5x + 25/4 = 13/4$
Step 3: Factor
$(x - 5/2)^2 = 13/4$
Step 4: Square root
$x - 5/2 = \pm\sqrt{(13/4)}$
$x - 5/2 = \pm\sqrt{13}/2$
Step 5: Solve
$x = 5/2 \pm \sqrt{13}/2$
$x = (5 \pm \sqrt{13})/2$
WHY IT WORKS:
Perfect square always ≥ 0
$(x + p)^2 \geq 0$
So if $(x + p)^2 = q$:
- $q > 0$: Two solutions
- $q = 0$: One solution
- $q < 0$: No real solutions

F. Quadratic Formula

Diagram 85: The Quadratic Formula

THE ULTIMATE FORMULA
(Works for ANY quadratic!)
For $ax^2 + bx + c = 0$:

$x = -b \pm \sqrt{(b^2 - 4ac)}$

$2a$

PARTS OF THE FORMULA:

$-b$

$\pm$

$\sqrt{b^2-4ac}$

└─ Denominator

└──────────── Discriminant

└──────────────── Plus-Minus

└──────────────────── Opposite of b

DISCRIMINANT (Δ):

$$\boxed{\Delta = b^2 - 4ac}$$

Tells us nature of roots:

$\Delta > 0$: Two distinct real roots

$\Delta = 0$: Two equal roots

↑

$\Delta < 0$: No real roots

\

USING THE FORMULA:

EXAMPLE 1: $x^2 + 5x + 6 = 0$
Identify: $a=1$, $b=5$, $c=6$
Step 1: Calculate discriminant
$\Delta = 5^2 - 4(1)(6)$
$\Delta = 25 - 24 = 1$
Step 2: Apply formula
$x = -5 \pm \sqrt{1}$
————————
$2(1)$
$x = -5 \pm 1$
———————
2
Step 3: Find both solutions
$x = -5+1 = -4 = -2$
—————
——
2
2
OR
$x = -5-1 = -6 = -3$
—————
——
2
2
Roots: $x = -2$ or $x = -3$

EXAMPLE 2: $2x^2 + 3x - 5 = 0$
Identify: $a=2$, $b=3$, $c=-5$
Step 1: Discriminant126
$\Delta = 3^2 - 4(2)(-5)$
$\Delta = 9 + 40 = 49$
Step 2: Formula
$x = -3 \pm \sqrt{49}$
—————————
$2(2)$
$x = -3 \pm 7$
———————
4
Step 3: Solutions
$x = -3+7 = 4 = 1$
—————
—
4
4
OR
$x = -3-7 = -10 = -5/2$

―――――
――――
4
4
Roots: x = 1 or x = -2.5
――――――――――――――――――――――――――――――――――
EXAMPLE 3: $x^2 - 4x + 4 = 0$
Identify: a=1, b=-4, c=4
Step 1: Discriminant
$\Delta = (-4)^2 - 4(1)(4)$
$\Delta = 16 - 16 = 0$
Equal roots! ($\Delta = 0$)
Step 2: Formula
$x = -(-4) \pm \sqrt{0}$
――――――――――
2(1)x = 4 ± 0
―――――
2
x = 4/2 = 2
Root: x = 2 (repeated)
――――――――――――――――――――――――――――――――――
EXAMPLE 4: $x^2 + 2x + 5 = 0$
Identify: a=1, b=2, c=5
Step 1: Discriminant
$\Delta = 2^2 - 4(1)(5)$
$\Delta = 4 - 20 = -16$
No real roots! ($\Delta < 0$)
Cannot take √-16 in real numbers
FORMULA MEMORY AID:
"Negative b,
Plus or minus square root,
b squared minus 4ac,
All over 2a"
Or picture:
-b
―――― ± √discriminant
2a
WHEN TO USE FORMULA:
✓ When factorization difficult
✓ When roots are irrational
✓ When completing square complex
✓ To verify other methods
✓ For quick solutions

Worked Examples with Complete Solutions:

Example 1: Solve $x^2 - 7x + 12 = 0$ by factorization

Diagram 86: Complete Factorization Solution

Given: $x^2 - 7x + 12 = 0$

STEP 1: Identify type

$a = 1$ (coefficient of x^2)

Simple trinomial

STEP 2: Find factors

Need two numbers that:

- Multiply to 12
- Add to -7

FACTOR SEARCH:

Factors of 12	Sum
1 × 12	13
2 × 6	8
3 × 4	7
-1 × -12	-13
-2 × -6	-8
-3 × -4	-7 ✓

Numbers: -3 and -4

STEP 3: Write factors

$x^2 - 7x + 12 = 0$

$(x - 3)(x - 4) = 0$

STEP 4: Apply zero product property

$x - 3 = 0$ OR $x - 4 = 0$

STEP 5: Solve

$x = 3$

OR x = 4129
VERIFICATION:
For x = 3:
3² - 7(3) + 12 = 9 - 21 + 12 = 0 ✓
For x = 4:
4² - 7(4) + 12 = 16 - 28 + 12 = 0 ✓
VISUAL REPRESENTATION:
12
/ \
-3
-4
└──┬──┘
-7
Area Model:
x
-4
┌───┬────┐
x │x² │-4x │
├───┼────┤
-3│-3x│ 12 │
└───┴────┘
-4x + -3x = -7x ✓
ANSWER: x = 3 or x = 4
Solutions
Example 2: Solve $2x^2 + 7x + 3 = 0$ by factorization
Diagram 87: Splitting Middle Term
Given: 2x² + 7x + 3 = 0
STEP 1: Identify coefficients
a = 2, b = 7, c = 3
STEP 2: Find a × c130
2 × 3 = 6
STEP 3: Find factors of 6 that add to 7
┌─────────────────────┐
│ Factors of 6 │
Sum
│
├────────────┼───────┤
│ 1 × 6
│
7 ✓ │
│ 2 × 3
│
5
│
│ -1 × -6

```
│
-7
│
│ -2 × -3
│
-5
│
└──────────────────────────┘
```

Use: 1 and 6

STEP 4: Split middle term

$2x^2 + 7x + 3 = 0$

$2x^2 + 1x + 6x + 3 = 0$

(rewrite 7x as 1x + 6x)

STEP 5: Factor by grouping

$2x^2 + 1x + 6x + 3 = 0$

Group 1: $2x^2 + 1x$

Factor: $x(2x + 1)$

Group 2: $6x + 3$

Factor: $3(2x + 1)$

Combined: $x(2x + 1) + 3(2x + 1) = 0$

STEP 6: Factor out common (2x + 1)

$(2x + 1)(x + 3) = 0$

STEP 7: Solve

$2x + 1 = 0$ OR $x + 3 = 0$

$2x = -1$

OR $x = -3$

$x = -1/2$

OR $x = -3$

VISUAL GROUPING:

```
2x² + 1x + 6x + 3131
└───┬───┘  └──┬──┘
x(2x+1) 3(2x+1)
└─────┬─────┘
(2x+1)(x+3)
```

VERIFICATION:

For $x = -1/2$:

$2(-1/2)^2 + 7(-1/2) + 3$

$= 2(1/4) - 7/2 + 3$

$= 1/2 - 7/2 + 6/2$

$= 0$ ✓

For $x = -3$:

$2(-3)^2 + 7(-3) + 3$

$= 2(9) - 21 + 3$

$= 18 - 21 + 3$

$= 0$ ✓

ANSWER: $x = -1/2$ or $x = -3$
DECIMAL: $x = -0.5$ or $x = -3$
Example 3: Solve $x^2 + 6x + 5 = 0$ by completing the square
Diagram 88: Completing Square Solution
Given: $x^2 + 6x + 5 = 0$
STEP 1: Move constant to right
$x^2 + 6x = -5$
STEP 2: Find value to complete square
Coefficient of $x = 6$
Half of $6 = 3$
Square it: $3^2 = 9$
VISUAL MODEL:
x
6

```
 ┌───┬────┐
x │x² │ 6x │
 └───┴────┘
```


Split 6x into two 3x:
x
3
3

```
 ┌───┬───┬───┐
x │x² │3x │3x │
 ├───┼───┼───┤
3 │3x │ 9 │
 │← Add 9
 └───┴───┴───┘
```

STEP 3: Add 9 to both sides
$x^2 + 6x + 9 = -5 + 9$
$x^2 + 6x + 9 = 4$
STEP 4: Factor left side
$(x + 3)^2 = 4$
STEP 5: Take square root
$\sqrt{(x + 3)^2} = \pm\sqrt{4}$
$x + 3 = \pm 2$
STEP 6: Solve both cases
Case 1: $x + 3 = 2$
$x = 2 - 3 = -1$
Case 2: $x + 3 = -2$
$x = -2 - 3 = -5$
STEP-BY-STEP VISUAL:
$x^2 + 6x + 5 = 0$
↓ (move 5)
$x^2 + 6x = -5$
↓ (add 9)
$x^2 + 6x + 9 = 4$

↓ (factor)
$(x + 3)^2 = 4$
↓ (√)
$x + 3 = \pm 2$
↓ (solve)
$x = -1$ or $x = -5$[133]
VERIFICATION:
For $x = -1$:
$(-1)^2 + 6(-1) + 5 = 1 - 6 + 5 = 0$ ✓
For $x = -5$:
$(-5)^2 + 6(-5) + 5 = 25 - 30 + 5 = 0$ ✓
NUMBER SQUARE VISUAL:
$(x + 3)^2 = 4$
2
┌───┐
2 │ 4 │ Area = 4
└───┘
Side length = 2
So $x + 3 = \pm 2$
ANSWER: $x = -1$ or $x = -5$
Example 4: Solve $3x^2 - 5x + 1 = 0$ using quadratic formula
Diagram 89: Quadratic Formula Solution
Given: $3x^2 - 5x + 1 = 0$
STEP 1: Identify coefficients

$a = 3$
$b = -5$
$c = 1$

STEP 2: Calculate discriminant
$\Delta = b^2 - 4ac$
$\Delta = (-5)^2 - 4(3)(1)$
$\Delta = 25 - 12$
$\Delta = 13$
Since $\Delta > 0$: Two distinct real roots[134]
STEP 3: Apply quadratic formula
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$
$$x = \frac{-(-5) \pm \sqrt{13}}{2(3)}$$

$$x = \frac{5 \pm \sqrt{13}}{6}$$

STEP 4: Find both solutions

$$x_1 = \frac{5 + \sqrt{13}}{6}$$

$$x_2 = \frac{5 - \sqrt{13}}{6}$$

STEP 5: Calculate decimal values

$\sqrt{13} \approx 3.606$

$x_1 = \frac{5 + 3.606}{6} = \frac{8.606}{6} \approx 1.43$

$x_2 = \frac{5 - 3.606}{6} = \frac{1.394}{6} \approx 0.23$

FORMULA BREAKDOWN:

$\frac{-b}{2a} \pm \sqrt{\Delta}$

$\frac{-(-5)}{2(3)} \pm \sqrt{13}$135

$\frac{5}{6} \pm \sqrt{13}$

DISCRIMINANT ANALYSIS:

$\Delta = 13 > 0$

↓

Two distinct real roots

↓

Roots are irrational ($\sqrt{13}$)

EXACT ANSWER:

$x = (5 + \sqrt{13})/6$ or $x = (5 - \sqrt{13})/6$

APPROXIMATE:

$x \approx 1.43$ or $x \approx 0.23$

VERIFICATION (using $x \approx 1.43$):

$3(1.43)^2 - 5(1.43) + 1$

$\approx 3(2.04) - 7.15 + 1$

$\approx 6.12 - 7.15 + 1$

```
≈ 0.03 ≈ 0 ✓ (close enough)
```

Example 5: Form a quadratic equation with roots 3 and -2

Diagram 90: Forming Equation from Roots

```
Given roots: α = 3, β = -2
METHOD 1: Using Factors
STEP 1: Write factors
If x = 3, then (x - 3) = 0136
If x = -2, then (x - (-2)) = 0
(x + 2) = 0
STEP 2: Multiply factors
(x - 3)(x + 2) = 0
STEP 3: Expand using FOIL
F: x × x = x²
O: x × 2 = 2x
I: -3 × x = -3x
L: -3 × 2 = -6
EXPANSION BOX:
x
+2
┌─────┬─────┐
x │ x² │+2x │
├─────┼─────┤
-3 │-3x │ -6 │
└─────┴─────┘
STEP 4: Combine terms
x² + 2x - 3x - 6 = 0
x² - x - 6 = 0
──────────────────────────────────
METHOD 2: Using Sum and Product
STEP 1: Calculate sum of roots
Sum = α + β = 3 + (-2) = 1
STEP 2: Calculate product of roots
Product = α × β = 3 × (-2) = -6
STEP 3: Use formula
x² - (sum)x + (product) = 0
x² - (1)x + (-6) = 0
x² - x - 6 = 0137
VERIFICATION:
For equation x² - x - 6 = 0
a = 1, b = -1, c = -6
Sum of roots = -b/a = -(-1)/1 = 1
= 3 + (-2) ✓
Product of roots = c/a = -6/1 = -6
= 3 × (-2) ✓
FACTORED FORM:
```

$x^2 - x - 6 = 0$
$(x - 3)(x + 2) = 0$
$x = 3$ or $x = -2$ ✓
VISUAL SUMMARY:
Roots: 3 and -2
↓
↓
Factors: $(x-3)(x+2)$
↓
Expand
↓
$x^2 - x - 6 = 0$
ANSWER: $x^2 - x - 6 = 0$

Example 6: The sum of roots of a quadratic equation is 5 and their product is 6. Find the equation and its roots.

Diagram 91: Complete Problem Solution

Given:
Sum of roots = 5
Product of roots = 6138
STEP 1: Form equation
Using: $x^2 - (\text{sum})x + (\text{product}) = 0$
$x^2 - 5x + 6 = 0$
STEP 2: Solve for roots (factorization)
Need two numbers that:
• Multiply to 6
• Add to 5

```
┌─────────────────────────┐
│ Factors of 6 │
Sum
│
├──────────────┼─────────┤
│ 1 × 6
│
7
│
│ 2 × 3
│
5 ✓ │
│ -1 × -6
│
-7
│
│ -2 × -3
│
-5
```

```
│
└─────────────────────────┘
```

Numbers: 2 and 3

STEP 3: Write factors

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

STEP 4: Solve

$x - 2 = 0$ OR $x - 3 = 0$

$x = 2$

OR $x = 3$

VERIFICATION:

Sum: $2 + 3 = 5$ ✓

Product: $2 \times 3 = 6$ ✓

VISUAL RELATIONSHIP:

Roots: 2 and 3

↓

↓

Sum = 5, Product = 6

↓

$x^2 - 5x + 6 = 0$139

↓

Factors: $(x-2)(x-3)$

↓

Solutions: $x=2$, $x=3$

AREA MODEL:

```
x
-3
┌───┬────┐
x │x² │-3x │
├───┼────┤
-2│-2x│ 6 │
└───┴────┘
```

$-3x + -2x = -5x$ ✓

Constant = 6 ✓

ANSWER:

Equation: $x^2 - 5x + 6 = 0$

Roots: $x = 2$ or $x = 3$

Roots (x-intercepts)

Class Activity:

Activity 1: Coefficient Hunt

Diagram 92: Identification Exercise

Identify a, b, c for each equation:

1. $2x^2 + 7x - 3 = 0$

```
┌──────────────────┐
│ a = 2
```

```
b = 7

c = -3
```

2. $x^2 - 4 = 0$

```
a = 1

b = 0 (no x)

c = -4
```

3. $3x^2 + 5x = 0$

```
a = 3

b = 5

c = 0 (no const)
```

4. $-x^2 + 3x + 2 = 0$

```
a = -1

b = 3

c = 2
```

Activity 2: Method Comparison
Solve $x^2 - 5x + 6 = 0$ using all three methods and compare:
1. Factorization
2. Completing the square
3. Quadratic formula

Activity 3: Real-World Problem The area of a rectangle is 24 cm^2. Its length is 2 cm more than its width. Find the dimensions.
Set up: $w(w + 2) = 24$ Solve: $w^2 + 2w - 24 = 0$

Evaluation Questions:
1. Identify the coefficients a, b, c in $3x^2 - 7x + 2 = 0$.
2. Form a quadratic equation with roots 4 and -1.
3. Solve $x^2 + 7x + 12 = 0$ by factorization.
4. Find the sum and product of roots of $2x^2 - 8x + 6 = 0$.
5. Solve $x^2 - 6x + 5 = 0$ by completing the square.

Assignment:

1. Solve by factorization: (a) $x^2 - 9x + 20 = 0$ (b) $2x^2 + 5x + 3 = 0$ (c) $x^2 - 25 = 0$
2. Solve by completing the square: (a) $x^2 + 8x + 7 = 0$ (b) $2x^2 - 12x + 10 = 0$
3. Solve using quadratic formula: (a) $x^2 - 3x - 5 = 0$ (b) $3x^2 + 2x - 4 = 0$
4. Form quadratic equations with these roots: (a) 5 and 2 (b) -3 and 4 (c) 1/2 and 3
5. The sum of a number and its reciprocal is 13/6. Find the number. (Hint: Let number be x, then $x + 1/x = 13/6$)

WEEK 7: QUADRATIC EQUATIONS II

Learning Objectives:

By the end of the lesson, students should be able to:

1. Form quadratic equations from given conditions
2. Find relationships between roots and coefficients
3. Verify given roots of quadratic equations
4. Solve word problems involving quadratic equations
5. Apply quadratic equations to real-life situations
6. Analyze the nature of roots using the discriminant

Entry Behaviour:

Students can solve quadratic equations by various methods and understand the relationship between roots and coefficients.

Instructional Materials:

- Whiteboard and markers
- Calculator
- Real-life scenario cards
- Graph paper
- Formula charts

Content:

A. Forming Equations from Given Conditions

Diagram 93: Types of Conditions

```
CONDITION TYPE 1: Given specific roots
Example: Roots are 3 and -4
Solution: (x - 3)(x + 4) = 0
x² + x - 12 = 0
```

```
CONDITION TYPE 2: Given sum and product
Example: Sum = 7, Product = 10
```

142143

```
Formula: x² - (sum)x + (product) = 0
Solution: x² - 7x + 10 = 0
```

```
CONDITION TYPE 3: Roots related to each other
Example: One root is twice the other
Let roots be α and 2α
Given additional info to find α
Then form equation
```

CONDITION TYPE 4: Roots differ by a constant
Example: Roots differ by 3
If roots are α and β:
$\beta - \alpha = 3$ or $\alpha - \beta = 3$
Use with sum/product to find values

DECISION TREE:

```
Given Condition
│
┌────┴────┐
│
│
Specific
Relationship
Roots
Between Roots
│
│
Use
Use algebra
Factors
to find roots
│
│
└────┬────┘
│
Form Equation144
```

Diagram 94: Complex Root Relationships

CASE 1: One root is k times the other
Let roots be α and $k\alpha$
Sum: $\alpha + k\alpha = \alpha(1 + k)$
Product: $\alpha \times k\alpha = k\alpha^2$
Example: One root is 3 times the other
Sum = 12
Let roots be α and 3α
$\alpha + 3\alpha = 12$
$4\alpha = 12$
$\alpha = 3$
Roots: 3 and 9
Equation: $(x - 3)(x - 9) = 0$
$x^2 - 12x + 27 = 0$
Sum = $4\alpha = 12$

CASE 2: Roots differ by a constant
Let roots be α and $\alpha + d$
(where d is the difference)

Sum: $\alpha + (\alpha + d) = 2\alpha + d$
Product: $\alpha(\alpha + d) = \alpha^2 + \alpha d$
Example: Roots differ by 4
Sum = 10145
Let roots be α and $\alpha + 4$
$\alpha + (\alpha + 4) = 10$
$2\alpha + 4 = 10$
$2\alpha = 6$
$\alpha = 3$
Roots: 3 and 7
Equation: $(x - 3)(x - 7) = 0$
$x^2 - 10x + 21 = 0$
NUMBER LINE:
3
7
●─────────●
└────4────┘
(difference)

CASE 3: Roots are reciprocals
If roots are α and $1/\alpha$
Product: $\alpha \times 1/\alpha = 1$
Example: Roots are reciprocals
Sum = 5/2
Let roots be α and $1/\alpha$
$\alpha + 1/\alpha = 5/2$
Multiply by α:
$\alpha^2 + 1 = 5\alpha/2$
$2\alpha^2 + 2 = 5\alpha$
$2\alpha^2 - 5\alpha + 2 = 0$
Solve: $(2\alpha - 1)(\alpha - 2) = 0$
$\alpha = 1/2$ or $\alpha = 2$
Roots: 1/2 and 2 (reciprocals ✓)146

CASE 4: Roots are opposites
If roots are α and $-\alpha$
Sum: $\alpha + (-\alpha) = 0$
Example: Roots are opposites
Product = -16
Let roots be α and $-\alpha$
$\alpha \times (-\alpha) = -16$
$-\alpha^2 = -16$
$\alpha^2 = 16$
$\alpha = \pm 4$
Roots: 4 and -4

Equation: $x^2 - 16 = 0$

VISUAL:

$-\alpha$

α

●——┼——●

0

↑

↑

Opposite on number line

CASE 5: Sum of squares given

If $\alpha^2 + \beta^2$ is known:

Use identity:

$(\alpha + \beta)^2 = \alpha^2 + 2\alpha\beta + \beta^2$

$\alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta$

Example: $\alpha^2 + \beta^2 = 13$

$\alpha + \beta = 5147$

$13 = 5^2 - 2\alpha\beta$

$13 = 25 - 2\alpha\beta$

$2\alpha\beta = 12$

$\alpha\beta = 6$

Sum = 5, Product = 6

Equation: $x^2 - 5x + 6 = 0$

FORMULA BOX:

$\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$\alpha^3 + \beta^3 = (\alpha+\beta)^3 - 3\alpha\beta(\alpha+\beta)$

$(\alpha-\beta)^2 = (\alpha+\beta)^2 - 4\alpha\beta$

B. Verification of Roots

Diagram 95: Root Verification Methods

METHOD 1: Direct Substitution

Given: $x^2 - 5x + 6 = 0$

Verify: $x = 2$ and $x = 3$ are roots

For $x = 2$:

$2^2 - 5(2) + 6$

$= 4 - 10 + 6$

$= 0$ ✓

For $x = 3$:

$3^2 - 5(3) + 6$

$= 9 - 15 + 6$

$= 0$ ✓

VERIFICATION CHECKLIST:

□ Substitute value
□ Follow order of operations
□ Should equal zero
□ Check both roots

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METHOD 2: Using Sum and Product
Given: $2x^2 + 5x - 3 = 0$
Verify: $x = 1/2$ and $x = -3$ are roots
From equation:
$a = 2,\ b = 5,\ c = -3$
Sum should be: $-b/a = -5/2$
Check: $1/2 + (-3) = 1/2 - 6/2 = -5/2$ ✓
Product should be: $c/a = -3/2$
Check: $(1/2) \times (-3) = -3/2$ ✓
FORMULA TABLE:

Given Equation	Check
$ax^2+bx+c = 0$	$\alpha+\beta = -b/a$ $\alpha\beta = c/a$

METHOD 3: Factorization Check
Given: $x^2 - 7x + 12 = 0$
Verify: Roots are 3 and 4
Factor: $(x - 3)(x - 4)$
Expand: $x^2 - 4x - 3x + 12$
$= x^2 - 7x + 12$ ✓
Matches original equation
VISUAL:
Equation → Factor → Roots
$x^2-7x+12$ $(x-3)(x-4)$ 3,4
Expand ← Check → Verify

METHOD 4: Graphical Verification
Plot $y = ax^2 + bx + c$
Roots are where graph crosses x-axis
Example: $y = x^2 - 5x + 6$
Crosses at $x = 2$ and $x = 3$ ✓
VERIFICATION ERRORS TO AVOID:
✗ Arithmetic mistakes

✗ Sign errors
✗ Forgetting negative roots
✗ Not checking both roots
✗ Calculation without showing work

C. Nature of Roots (Discriminant)

Diagram 96: Discriminant Analysis

THE DISCRIMINANT (Δ)
For $ax^2 + bx + c = 0$:

$$\boxed{\Delta = b^2 - 4ac}$$

DETERMINES:150
• Number of roots
• Type of roots
• Nature of roots

CASE 1: $\Delta > 0$
Two distinct real roots
Example: $x^2 - 5x + 6 = 0$
$a = 1, b = -5, c = 6$
$\Delta = (-5)^2 - 4(1)(6)$
$\Delta = 25 - 24 = 1 > 0$
Two crossings

CASE 2: $\Delta = 0$
Two equal roots (one repeated root)
Example: $x^2 - 4x + 4 = 0$
$a = 1, b = -4, c = 4$
$\Delta = (-4)^2 - 4(1)(4)$
$\Delta = 16 - 16 = 0$
One touching point

CASE 3: $\Delta < 0$
No real roots (complex roots)
Example: $x^2 + 2x + 5 = 0$
$a = 1, b = 2, c = 5$
$\Delta = 2^2 - 4(1)(5)$
$\Delta = 4 - 20 = -16 < 0$
No crossings

SPECIAL CASE: Perfect Square
When $\Delta = k^2$ (perfect square)
Roots are rational
Example: $2x^2 + 7x + 3 = 0$

$\Delta = 7^2 - 4(2)(3)$
$\Delta = 49 - 24 = 25 = 5^2$
Roots will be rational:
$x = (-7 \pm 5)/4$
$x = -1/2$ or $x = -3$
DISCRIMINANT TABLE:

Δ

Nature

> 0
2 distinct real
$= 0$
2 equal roots
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< 0
No real roots

$= k^2$
Rational roots
$\neq k^2$
Irrational roots

2 real 1 real No real
distinct equal roots
roots root
VISUAL SUMMARY:
Δ > 0: /___/___
Crosses twice
Δ = 0: /___•___
Touches once
Δ < 0: /______
Never crosses

D. Word Problems with Quadratic Equations

Diagram 97: Problem-Solving Framework

STEPS FOR WORD PROBLEMS:
1. READ carefully
2. IDENTIFY what to find
3. DEFINE variables
4. SET UP equation
5. SOLVE equation
6. CHECK answer
7. STATE conclusion

PROBLEM TYPES:[153]

1. NUMBER PROBLEMS
2. AGE PROBLEMS
3. AREA/PERIMETER
4. MOTION/DISTANCE
5. WORK/TIME
6. BUSINESS/PROFIT

TRANSLATION GUIDE:
"sum of" → +
"product of" → ×
"difference of" → -
"quotient of" → ÷
"is/equals" → =
"more than" → +
"less than" → -
"times" → ×
"squared" → ²

Diagram 98: Number Problems

PROBLEM TYPE 1: Number and Square
Problem: The square of a number exceeds the number by 12. Find the number.
STEP 1: Define variable
Let the number be x
STEP 2: Set up equation
"square exceeds number by 12"
$x^2 = x + 12$
STEP 3: Rearrange to standard form
$x^2 - x - 12 = 0$
STEP 4: Solve by factorization
Need factors of -12 that add to -1
Factors: -4 and 3[154]
$(x - 4)(x + 3) = 0$
$x = 4$ or $x = -3$
STEP 5: Check both solutions
For $x = 4$:
$4^2 = 16$, $4 + 12 = 16$ ✓
For $x = -3$:

$(-3)^2 = 9$, $-3 + 12 = 9$ ✓

STEP 6: State answer

The number is 4 or -3

VISUAL:

Number: x

Square: x^2

Exceeds by 12: $x^2 = x + 12$

x

x+12

●———●————●

x²

PROBLEM TYPE 2: Sum and Product

Problem: Two numbers differ by 3 and their product is 70. Find them.

STEP 1: Define variables

Let smaller number be x

Larger number is $x + 3$

STEP 2: Set up equation

"product is 70"

$x(x + 3) = 70$

STEP 3: Expand and rearrange

$x^2 + 3x = 70$155

$x^2 + 3x - 70 = 0$

STEP 4: Solve

$(x + 10)(x - 7) = 0$

$x = -10$ or $x = 7$

STEP 5: Find both numbers

If $x = 7$:

Numbers are 7 and 10

Check: $10 - 7 = 3$ ✓

$7 \times 10 = 70$ ✓

If $x = -10$:

Numbers are -10 and -7

Check: $-7 - (-10) = 3$ ✓

$-10 \times -7 = 70$ ✓

ANSWER: The numbers are 7 and 10, or -10 and -7

│

●————●

└——3———┘

└—3—┘

Diagram 99: Area Problems

PROBLEM: Rectangle Area

Problem: A rectangle has length 3 cm more

than its width. Its area is 40 cm².
Find the dimensions.
STEP 1: Define variable
Let width = w cm
Length = $(w + 3)$ cm
VISUAL:

```
┌───────────────┐
│
│ w
│
│└───────────────┘
w + 3
```

STEP 2: Set up equation
Area = length × width
$40 = (w + 3) \times w$
$40 = w^2 + 3w$
STEP 3: Standard form
$w^2 + 3w - 40 = 0$
STEP 4: Factorize
$(w + 8)(w - 5) = 0$
$w = -8$ or $w = 5$
STEP 5: Choose valid solution
Width cannot be negative
$w = 5$ cm
Length = $5 + 3 = 8$ cm
STEP 6: Verify
Area = $8 \times 5 = 40$ cm² ✓
ANSWER: Width = 5 cm, Length = 8 cm
AREA MODEL:

```
┌─────────┐
5 │////////│ 8
│////////│
└─────────┘
```

Area = 40 cm²

PROBLEM: Triangle Area
Problem: The base of a triangle is 4 cm
longer than its height. If the
area is 30 cm², find base and height.
156157
STEP 1: Define variables
Let height = h cm
Base = $(h + 4)$ cm
$h+4$
STEP 2: Use area formula
Area = ½ × base × height

$30 = \frac{1}{2} \times (h + 4) \times h$
$60 = h\,(h + 4)$
$60 = h^2 + 4h$
STEP 3: Standard form
$h^2 + 4h - 60 = 0$
STEP 4: Factorize
$(h + 10)\,(h - 6) = 0$
$h = -10$ or $h = 6$
STEP 5: Valid solution
Height cannot be negative
$h = 6$ cm
Base = 6 + 4 = 10 cm
STEP 6: Verify
Area = $\frac{1}{2} \times 10 \times 6 = 30$ cm^2 ✓
ANSWER: Height = 6 cm, Base = 10 cm

Diagram 100: Motion Problems

PROBLEM: Speed and Distance
Problem: A car travels 180 km. If the
speed was 15 km/h faster, the
journey would take 1 hour less. Find the original speed.
STEP 1: Define variable
Let original speed = v km/h
New speed = (v + 15) km/h
STEP 2: Use Time = Distance/Speed
Original time = 180/v hours
New time = 180/(v+15) hours
STEP 3: Set up equation
"1 hour less"
$180/v - 180/(v+15) = 1$
STEP 4: Solve
Multiply by v(v+15):
$180(v+15) - 180v = v(v+15)$
$180v + 2700 - 180v = v^2 + 15v$
$2700 = v^2 + 15v$
$v^2 + 15v - 2700 = 0$
STEP 5: Use quadratic formula
$v = (-15 \pm \sqrt{(225 + 10800)})/2$
$v = (-15 \pm \sqrt{11025})/2$
$v = (-15 \pm 105)/2$
$v = 45$ or $v = -60$
STEP 6: Valid solution
Speed cannot be negative
v = 45 km/h
VERIFICATION:
Original time: 180/45 = 4 hours
New time: 180/60 = 3 hours

Difference: 4 - 3 = 1 hour ✓
ANSWER: Original speed = 45 km/h
TIME-SPEED TABLE:

158159

Time │Speed │

Original
180/v │ v

New
180/ │ v+15 │

(v+15) │

Difference │ 1
15 │

Worked Examples with Complete Solutions:

Example 1: One root of $2x^2 + kx - 6 = 0$ is 2. Find k and the other root.

Diagram 101: Finding Unknown Coefficient

Given: $2x^2 + kx - 6 = 0$
One root is x = 2
METHOD 1: Direct Substitution
STEP 1: Substitute x = 2
$2(2)^2 + k(2) - 6 = 0$
$2(4) + 2k - 6 = 0$
$8 + 2k - 6 = 0$
$2k + 2 = 0$
$2k = -2$
$k = -1$
STEP 2: Find other root
Equation becomes: $2x^2 - x - 6 = 0$
Using sum of roots:
$\alpha + \beta = -b/a = -(-1)/2 = 1/2$
Since $\alpha = 2$:
$2 + \beta = 1/2$
$\beta = 1/2 - 2 = -3/2$

METHOD 2: Using Product of Roots STEP 1: Use product formula
$\alpha \times \beta = c/a = -6/2 = -3$
Since $\alpha = 2$:
$2 \times \beta = -3$
$\beta = -3/2$
STEP 2: Use sum to find k

$\alpha + \beta = -k/2$
$2 + (-3/2) = -k/2$
$1/2 = -k/2$
$k = -1$
VERIFICATION:
Equation: $2x^2 - x - 6 = 0$
For $x = 2$:
$2(2)^2 - 2 - 6 = 8 - 2 - 6 = 0$ ✓
For $x = -3/2$:
$2(-3/2)^2 - (-3/2) - 6$
$= 2(9/4) + 3/2 - 6$
$= 9/2 + 3/2 - 12/2$
$= 0$ ✓
VISUAL SUMMARY:
$2x^2 + kx - 6 = 0$
↓ $(x=2)$
$8 + 2k - 6 = 0$
↓
$k = -1$
↓
$2x^2 - x - 6 = 0$
↓
Roots: 2 and $-3/2$
ANSWER: $k = -1$, other root $= -3/2$
NUMBER LINE:
────┼─────┼─────┼─────┼────→
160161
-2
-3/2
0
2
↑
↑
Root 2
Root 1
Example 2: The roots of $x^2 - px + 12 = 0$ are in the ratio 3:4. Find p and the roots.
Diagram 102: Ratio Root Problem
Given: $x^2 - px + 12 = 0$
Roots in ratio 3:4
STEP 1: Express roots
Let roots be $3k$ and $4k$
(where k is a constant)
RATIO VISUAL:
$3k$
$4k$

●●●
●●●●

Ratio 3:4

STEP 2: Use product of roots

Product = $c/a = 12/1 = 12$

$3k \times 4k = 12$

$12k^2 = 12$

$k^2 = 1$

$k = \pm 1$

STEP 3: Find actual roots

If $k = 1$:

Roots are 3 and 4

If $k = -1$:

Roots are -3 and -4

STEP 4: Find p using sum

Case 1: Roots are 3 and 4

Sum = $-(-p)/1 = p$162

$3 + 4 = p$

$p = 7$

Case 2: Roots are -3 and -4

Sum = p

$-3 + (-4) = p$

$p = -7$

VERIFICATION:

For $p = 7$: $x^2 - 7x + 12 = 0$

Factor: $(x - 3)(x - 4) = 0$

Roots: 3 and 4

Ratio: 3:4 ✓

Product: $3 \times 4 = 12$ ✓

For $p = -7$: $x^2 + 7x + 12 = 0$

Factor: $(x + 3)(x + 4) = 0$

Roots: -3 and -4

Ratio: -3:-4 = 3:4 ✓

Product: $(-3) \times (-4) = 12$ ✓

COMPLETE SOLUTION TABLE:

p	Roots	Check
7	3, 4	3:4 ✓
-7	-3, -4	3:4 ✓

└──────┴─────────┴───────────┘
ANSWER:
p = 7 with roots 3 and 4, OR
p = -7 with roots -3 and -4
VISUAL RATIO:
Positive case:
├───┼────┤163
3
4
3 parts: 4 parts
Negative case:
├────┼───┤
-4 -3
4 parts: 3 parts (same ratio)

Example 3: The perimeter of a rectangle is 26 cm and its area is 40 cm^2. Find its dimensions.

Diagram 103: Rectangle Problem

Given:
Perimeter = 26 cm
Area = 40 cm^2
STEP 1: Define variables
Let length = l cm
Let width = w cm
RECTANGLE VISUAL:
┌───────────────┐
│
│ w
│
└───────────────┘
l
STEP 2: Set up equations
Perimeter: 2(l + w) = 26
l + w = 13 ... (1)
Area: l × w = 40 ... (2)
STEP 3: Express l in terms of w
From (1): l = 13 - w
STEP 4: Substitute into (2)
(13 - w) × w = 40
$13w - w^2 = 40$
$-w^2 + 13w - 40 = 0$
$w^2 - 13w + 40 = 0$164
STEP 5: Factorize
Need factors of 40 that add to -13
Factors: -8 and -5
(w - 8) (w - 5) = 0
w = 8 or w = 5

STEP 6: Find corresponding lengths
If w = 8: l = 13 - 8 = 5
If w = 5: l = 13 - 5 = 8
STEP 7: Interpret result
Dimensions are 8 cm × 5 cm
(Same rectangle, just swapping
length and width)
VERIFICATION:
Perimeter: 2(8 + 5) = 26 ✓
Area: 8 × 5 = 40 ✓
SOLUTION DIAGRAM:

```
┌────────────────┐
5 │///////////////│ 8
│////////////////│
└────────────────┘
8
```

Perimeter = 2(8+5) = 26 cm
Area = 8 × 5 = 40 cm²
DIMENSIONS TABLE:

Length	Width	Perimeter	Area
8	5	26	40
5	8	26	40

ANSWER: Length = 8 cm, Width = 5 cm165
(or vice versa)
ALGEBRAIC SOLUTION PATH:
l + w = 13
l × w = 40
↓
l = 13 - w
↓

$(13-w) \times w = 40$
↓
$w^2 - 13w + 40 = 0$
↓
$(w-8)(w-5) = 0$
↓
$w = 8$ or 5
↓
Dimensions: 8×5 cm

Example 4: A man is 24 years older than his son. In 4 years, the product of their ages will be 360. Find their present ages.

Diagram 104: Age Problem

Given:
Man is 24 years older than son
In 4 years: product of ages = 360
STEP 1: Define variables
Let son's present age = x years
Man's present age = (x + 24) years
AGE TABLE NOW:

Son	Man
x	x + 24

AGE TABLE IN 4 YEARS:166

Son	Man
x + 4	x + 28

STEP 2: Set up equation
"In 4 years, product = 360"

$(x + 4) (x + 28) = 360$
STEP 3: Expand
$x^2 + 28x + 4x + 112 = 360$
$x^2 + 32x + 112 = 360$
STEP 4: Standard form
$x^2 + 32x + 112 - 360 = 0$
$x^2 + 32x - 248 = 0$
STEP 5: Use quadratic formula
$a = 1, b = 32, c = -248$
$x = (-32 \pm \sqrt{(32^2 - 4(1)(-248))})/2$
$x = (-32 \pm \sqrt{(1024 + 992)})/2$
$x = (-32 \pm \sqrt{2016})/2$
$x = (-32 \pm 44.9)/2$
$x = 12.9/2$ or $-76.9/2$
$x \approx 6.45$ or $x \approx -38.45$
STEP 6: Choose valid solution
Age cannot be negative
$x \approx 6$ years (rounding to integer)
Actually, let's solve more carefully:
$x^2 + 32x - 248 = 0$
Try factoring:
$(x + 38) (x - 6) = 0$?
Check: $38 \times -6 = -228$ ✗ Let's try: $(x - 6) (x + ?)$
Need: $-6 \times ? = -248$
$? = 248/6 \approx 41.33$
Not exact factors, use formula:
$x = (-32 \pm \sqrt{2016})/2$
But $\sqrt{2016} = \sqrt{(16\times126)} = 4\sqrt{126}$
Actually, checking if 248 factors:
$248 = 8 \times 31$
Trying $(x - 6) (x + 38)$:
$x^2 + 38x - 6x - 228$
$= x^2 + 32x - 228$ ✗
Let me recalculate discriminant:
$\Delta = 1024 + 992 = 2016$
Since not perfect square,
use approximation:
$x \approx 6$ years
STEP 7: Find man's age
Man's age = 6 + 24 = 30 years
VERIFICATION (approximate):
In 4 years:
Son: 6 + 4 = 10 years
Man: 30 + 4 = 34 years
Product: $10 \times 34 = 340 \approx 360$

(Close enough with rounding)
For exact solution:
If $(x+4)(x+28) = 360$
$x^2 + 32x + 112 = 360$
$x^2 + 32x - 248 = 0$
Using completing square:
$x^2 + 32x = 248$
167168
$x^2 + 32x + 256 = 504$
$(x + 16)^2 = 504$
$x + 16 = \pm\sqrt{504}$
$x = -16 \pm \sqrt{504}$
$x = -16 + 22.45 = 6.45$
$x \approx 6$ years
TIMELINE VISUAL:
Now
In 4 years
Son: 6 ⟶ 10
↕24
↕24
Man: 30 ⟶ 34
Product in 4 years: $10 \times 34 = 340$
(Note: Slight discrepancy due to
rounding. Exact answer requires
keeping $\sqrt{504}$ form)
ANSWER:
Son's present age ≈ 6 years
Man's present age ≈ 30 years
Or exactly:
Son: $-16 + \sqrt{504} \approx 6.45$ years
Man: $8 + \sqrt{504} \approx 30.45$ years

Example 5: Determine the nature of roots of $3x^2 - 5x + 4 = 0$ without solving.

Diagram 105: Discriminant Analysis

Given: $3x^2 - 5x + 4 = 0$
STEP 1: Identify coefficients
$a = 3$
$b = -5$
$c = 4$
COEFFICIENT BOX:

a = 3 (x^2 term) 169
b = -5 (x term)
c = 4 (const)

STEP 2: Calculate discriminant

$\Delta = b^2 - 4ac$
$\Delta = (-5)^2 - 4(3)(4)$
$\Delta = 25 - 48$
$\Delta = -23$
CALCULATION BREAKDOWN:
$b^2 = (-5)^2 = 25$
$4ac = 4 \times 3 \times 4 = 48$
$\Delta = 25 - 48 = -23$
STEP 3: Analyze result
$\Delta = -23 < 0$
DISCRIMINANT DECISION:
$\Delta < 0$
↓
NO REAL ROOTS
↓
Complex/Imaginary
roots
STEP 4: Graphical interpretation
Since $\Delta < 0$, the parabola does
not cross the x-axis
│
Never crosses x-axis
NATURE OF ROOTS:

- Two complex conjugate roots
- Form: $p \pm qi$
- No real solutions
- Parabola entirely above

or below x-axis

COMPARISON TABLE:

Δ

Value
Nature

> 0
Positive │ 2 real

```
│ = 0
│
Zero
│ 1 real
│
│ < 0
│ Negative │ No real ✓ │
└─────────┴──────────┴───────────┘
```

ANSWER:
The equation has NO REAL ROOTS
(two complex conjugate roots)
Because: $\Delta = -23 < 0$
VISUAL SUMMARY:

```
3x² - 5x + 4 = 0
↓
Δ = -23
↓
Δ < 0
↓
No real roots
↓
/\
/ \ (graph above x-axis)
/____\
```

Class Activity:

Activity 1: Root Finding Challenge Diagram 106: Practice Problems

Find the value of k in each case:
1. $x^2 + kx + 9 = 0$ has equal roots
Hint: $\Delta = 0$
Solution:
$\Delta = k^2 - 4(1)(9) = 0$
$k^2 - 36 = 0$
$k = \pm 6$
2. $kx^2 - 6x + 2 = 0$ has one root as 2
Solution:
$k(2)^2 - 6(2) + 2 = 0$
$4k - 12 + 2 = 0$
$4k = 10$
$k = 5/2$
3. $2x^2 + 5x + k = 0$ has product of roots as 3
Solution:
Product $= c/a = k/2$
$k/2 = 3$
$k = 6$

Activity 2: Word Problem Workshop

In groups, create and solve word problems involving:
• Age relationships
• Area and perimeter
• Number relationships
• Speed and distance

Activity 3: Discriminant Detective

Without solving, determine the nature of roots:

1. $x^2 - 4x + 4 = 0$ ($\Delta = 0$, equal roots)
1712. $2x^2 + 3x - 5 = 0$ ($\Delta = 49 > 0$, 2 real)
3. $x^2 + x + 1 = 0$ ($\Delta = -3 < 0$, no real)

Evaluation Questions:

1. One root of $x^2 + px + 8 = 0$ is 4. Find p and the other root.
2. The roots of $2x^2 - 7x + k = 0$ are equal. Find k.
3. Form an equation whose roots are 5 and -3.
4. The sum of two numbers is 12 and their product is 35. Find the numbers.
5. Without solving, determine the nature of roots of $3x^2 + 7x + 5 = 0$.

Assignment:

1. If one root of $3x^2 + kx - 2 = 0$ is 2, find: (a) The value of k (b) The other root
2. The roots of $x^2 - px + 15 = 0$ are in the ratio 2:3. Find: (a) The value of p (b) The roots
3. A rectangle has length 5 cm more than its width. If the area is 84 cm², find the dimensions.
4. The product of two consecutive positive integers is 132. Find the integers.
5. Show that the equation $2x^2 - 5x + 4 = 0$ has no real roots. What is the nature of its roots?

WEEK 8: SIMULTANEOUS EQUATIONS

Learning Objectives:

By the end of the lesson, students should be able to:

1. Define simultaneous equations
2. Solve simultaneous linear equations by elimination method
3. Solve simultaneous equations by substitution method
4. Solve one linear and one quadratic equation simultaneously
5. Apply simultaneous equations to real-life problems
6. Choose the most appropriate method for different types

Entry Behaviour:

Students can solve linear equations, expand brackets, and perform basic algebraic manipulations.

Instructional Materials:

• Whiteboard and markers
• Algebra tiles or manipulatives
• Graph paper
• Calculator
• Worked examples charts
• Practice worksheets

Content:

A. Introduction to Simultaneous Equations

Definition: Simultaneous equations are two or more equations that share common variables and must be solved together to find values that satisfy all equations at once.

Diagram 107: Understanding Simultaneous Equations

WHAT ARE SIMULTANEOUS EQUATIONS?
Two equations with two unknowns (x and y):

```
Equation 1: ax + by = c
Equation 2: dx + ey = f
```

Solution: Values of x and y that make
BOTH equations true
VISUAL CONCEPT:
Equation 1: $2x + y = 7$
Points on line: (0,7), (1,5), (2,3), (3,1)
Equation 2: $x - y = 2$
Points on line: (0, -2), (1, -1), (2,0), (3,1)
Intersection point (3, 1) is the solution!
$x = 3$, $y = 1$ satisfies BOTH equations
VERIFICATION:
Equation 1: $2(3) + 1 = 6 + 1 = 7$ ✓
Equation 2: $3 - 1 = 2$ ✓

TYPES OF SOLUTIONS:
1. ONE UNIQUE SOLUTION
2. NO SOLUTION
Parallel lines (never meet)
═══════

═══════

3. INFINITE SOLUTIONS
Same line (coincident)
═══════ (both equations
represent same line)
STANDARD FORMS:
Linear-Linear:

```
ax + by = c

dx + ey = f
```

Linear-Quadratic:

```
y = mx + c

y = ax² + bx + c
```

B. Elimination Method

Diagram 108: Elimination Method Steps

PRINCIPLE: Eliminate one variable by making coefficients equal

GENERAL STEPS:176

```
┌─────────────────────────────────┐
 1. Write equations clearly
 2. Make coefficients equal
 3. Add or subtract equations
 4. Solve for one variable
 5. Substitute back
 6. Find other variable
 7. Verify solution
└─────────────────────────────────┘
```

STRATEGY GUIDE:

Look at coefficients:

```
┌───────────────────────────────┐
 Same coefficient already?
 → Directly add/subtract

 Different coefficients?
 → Multiply to make equal

 One coefficient is 1?
 → Good candidate for subst. │
└───────────────────────────────┘
```

SIGN RULES:

• Same signs → Subtract equations
• Different signs → Add equations

To eliminate:

+ with - → ADD
+ with + → SUBTRACT
- with - → SUBTRACT

- with + → ADD

Diagram 109: Elimination - Simple Case

EXAMPLE: Coefficients already equal
Solve: 2x + 3y = 13 ... (1)
2x - y = 5
... (2)177
STEP 1: Identify equal coefficients
Both have 2x
VISUAL:
Eq 1: 2x + 3y = 13
Eq 2: 2x - y = 5
↑↑
Same coefficient!
STEP 2: Subtract to eliminate x
(1) - (2):
2x + 3y = 13
-(2x - y = 5)
———————————
0 + 4y = 8
SUBTRACTION BREAKDOWN:

```
┌──────────────────────────┐
 2x - 2x = 0

 3y - (-y) = 3y + y = 4y │
│13 - 5 = 8

└──────────────────────────┘
```

STEP 3: Solve for y
4y = 8
y = 2
STEP 4: Substitute y = 2 into (1)
2x + 3(2) = 13
2x + 6 = 13
2x = 7
x = 7/2 = 3.5
SOLUTION BOX:

```
┌──────────────┐
 x = 3.5

 y = 2

└──────────────┘
```

VERIFICATION:
Equation 1: 2(3.5) + 3(2) = 7 + 6 = 13 ✓
Equation 2: 2(3.5) - 2 = 7 - 2 = 5 ✓
VISUAL SOLUTION PROCESS:

2x + 3y = 13
2x - y = 5
↓ (subtract)
4y = 8
↓ (÷4)
y = 2
↓ (substitute)
2x + 6 = 13
↓
x = 3.5

Diagram 110: Elimination - Multiplying First

EXAMPLE: Making coefficients equal
Solve: 3x + 2y = 16 ... (1)
2x + 5y = 21 ... (2)
STEP 1: Choose variable to eliminate
Let's eliminate x
STEP 2: Find LCM of x-coefficients
Coefficients: 3 and 2
LCM = 6
MULTIPLICATION PLAN:

Eq 1 × 2 → 6x + 4y = 32
Eq 2 × 3 → 6x + 15y = 63

STEP 3: Multiply equations
(1) × 2: 6x + 4y = 32 ... (3)
(2) × 3: 6x + 15y = 63 ... (4)
STEP 4: Subtract to eliminate x
178179
(4) - (3):
6x + 15y = 63
-(6x + 4y = 32)

0 + 11y = 31
SUBTRACTION VISUAL:
6x + 15y = 63
-(6x + 4y = 32)

0 + 11y = 31
STEP 5: Solve for y
11y = 31
y = 31/11
STEP 6: Substitute into original (1)
3x + 2(31/11) = 16
3x + 62/11 = 16
3x = 16 - 62/11

3x = (176 - 62)/11
3x = 114/11
x = 114/33 = 38/11
EXACT ANSWER:

x = 38/11

y = 31/11

DECIMAL APPROXIMATION:

x ≈ 3.45

y ≈ 2.82

VERIFICATION (using decimals):
Eq 1: 3(3.45) + 2(2.82) ≈ 10.35 + 5.64 = 15.99 ≈ 16 ✓
Eq 2: 2(3.45) + 5(2.82) ≈ 6.90 + 14.10 = 21.00 ✓180
SOLUTION PATH:
3x + 2y = 16
2x + 5y = 21
↓ (×2, ×3)
6x + 4y = 32
6x + 15y = 63
↓ (subtract)
11y = 31
↓ (÷11)
y = 31/11
↓ (substitute)
x = 38/11

C. Substitution Method

Diagram 111: Substitution Method Steps

PRINCIPLE: Express one variable in terms
of the other, then substitute
GENERAL STEPS:

1. Choose simpler equation

2. Express one variable

3. Substitute into other eq.

4. Solve resulting equation

5. Find other variable

6. Verify solution

WHEN TO USE SUBSTITUTION:
✓ One equation already solved for a variable
✓ Coefficient of 1 present
✓ One equation easily rearranged
✓ Non-linear equations (one quadratic)181
DECISION GUIDE:
Look at equations

Coefficient
All different
of 1?
coefficients?

↓
↓
Substitution Elimination
better
better

Diagram 112: Substitution - Basic Example

EXAMPLE: Simple substitution
Solve: y = 2x - 1
... (1)
3x + 2y = 12 ... (2)
STEP 1: Identify substitution
Equation (1) already has y isolated!
y = 2x - 1
STEP 2: Substitute into equation (2)
3x + 2(2x - 1) = 12
SUBSTITUTION VISUAL:
3x + 2y = 12
↓
3x + 2(2x-1) = 12

Replace y
STEP 3: Expand and simplify

3x + 4x - 2 = 12
7x - 2 = 12
7x = 14
x = 2
STEP 4: Find y using equation (1)
y = 2(2) - 1182
y = 4 - 1
y = 3
SOLUTION:

┌──────────────┐
x = 2

y = 3
└──────────────┘

VERIFICATION:
Eq 1: y = 2(2) - 1 = 3 ✓
Eq 2: 3(2) + 2(3) = 6 + 6 = 12 ✓
SOLUTION FLOW:
y = 2x - 1
3x + 2y = 12
↓
3x + 2(2x-1) = 12
↓
7x = 14
↓
x = 2
↓
y = 2(2) - 1
↓
y = 3
NUMBER LINE CHECK:
For x = 2, y = 3:
x: ───┼─────┼────→
0
2
y: ───┼─────┼─────┼────→
0
3

Diagram 113: Substitution - Rearranging First

EXAMPLE: Need to rearrange first
Solve: 2x + y = 7
... (1)
3x - 2y = 4 ... (2) STEP 1: Choose equation to rearrange
Equation (1) is simpler

(y has coefficient 1)
STEP 2: Express y in terms of x
From (1): $2x + y = 7$
$y = 7 - 2x$ ... (3)
REARRANGEMENT VISUAL:
$2x + y = 7$
↓ (-2x)
$y = 7 - 2x$
STEP 3: Substitute into equation (2)
$3x - 2(7 - 2x) = 4$
STEP 4: Expand brackets
$3x - 14 + 4x = 4$
EXPANSION BOX:
$-2(7 - 2x)$
$-2\times 7 + -2\times(-2x)$
$-14 + 4x$
STEP 5: Simplify and solve
$7x - 14 = 4$
$7x = 18$
$x = 18/7$
STEP 6: Find y using equation (3)
$y = 7 - 2(18/7)$
$y = 7 - 36/7$
$y = (49 - 36)/7$
$y = 13/7$
183184
EXACT SOLUTION:

$x = 18/7$

$y = 13/7$

DECIMAL FORM:

$x \approx 2.57$

$y \approx 1.86$

VERIFICATION:
Eq 1: $2(18/7) + 13/7 = 36/7 + 13/7 = 49/7 = 7$ ✓
Eq 2: $3(18/7) - 2(13/7) = 54/7 - 26/7 = 28/7 = 4$ ✓
STEP-BY-STEP PATH:
$2x + y = 7$

↓
y = 7 - 2x
↓ (substitute)
3x - 2(7-2x) = 4
↓ (expand)
3x - 14 + 4x = 4
↓ (simplify)
7x = 18
↓
x = 18/7
↓ (substitute back)
y = 7 - 2(18/7)
↓
y = 13/7

*D. Solving One Linear and One Quadratic*185

Diagram 114: Linear-Quadratic Systems

SYSTEM TYPE:

```
┌─────────────────────────┐
│ Linear: y = mx + c
│
│ Quadratic: y = ax²+bx+c │
└─────────────────────────┘
```

KEY POINTS:
• Can have 0, 1, or 2 solutions
• Use substitution method
• Leads to quadratic equation

POSSIBLE OUTCOMES:

1. TWO SOLUTIONS
Line intersects parabola twice
Point 1 Point 2
2. ONE SOLUTION
Line touches parabola (tangent)
Touch point
3. NO SOLUTION
Line misses parabola
Line doesn't touch
Linear line
Two intersection points = 2 solutions

Diagram 115: Solving Linear-Quadratic Example

EXAMPLE: Line and parabola
Solve: y = x + 2
... (1) [Linear]
y = x² - 4x + 6 ... (2) [Quadratic]
STEP 1: Substitute (1) into (2)
Since both equal y:
x + 2 = x² - 4x + 6

SUBSTITUTION VISUAL:
$y = x + 2$
$y = x^2 - 4x + 6$
↓ (both equal y)
$x + 2 = x^2 - 4x + 6$
STEP 2: Rearrange to standard form
$0 = x^2 - 4x + 6 - x - 2$
$0 = x^2 - 5x + 4$
$x^2 - 5x + 4 = 0$187
STEP 3: Factorize
Need factors of 4 that add to -5
Factors: -4 and -1
$(x - 4)(x - 1) = 0$
FACTOR CHECK:
4
/ \
-4 -1
└─┬─┘
-5 (sum)
STEP 4: Solve for x
$x - 4 = 0$ OR $x - 1 = 0$
$x = 4$
OR $x = 1$
STEP 5: Find corresponding y values
For $x = 4$:
$y = 4 + 2 = 6$
For $x = 1$:
$y = 1 + 2 = 3$
SOLUTIONS:

Solution 1: (4, 6)

Solution 2: (1, 3)

VERIFICATION:
Solution 1: (4, 6)
Eq 1: $y = 4 + 2 = 6$ ✓
Eq 2: $y = 16 - 16 + 6 = 6$ ✓
Solution 2: (1, 3)
Eq 1: $y = 1 + 2 = 3$ ✓
Eq 2: $y = 1 - 4 + 6 = 3$ ✓188
GRAPHICAL CHECK:
y
|

```
/\
6 |
/ \
|
/ ● \ (4,6)
5 | /
\
4 | /
\
3 |/ ●————————\ (1,3)
2 |/___________\
1 |
└————————————————— x
0 1 2 3 4 5
Line y = x + 2 intersects
parabola at (1,3) and (4,6)
SOLUTION PATH:
y = x + 2
y = x² - 4x + 6
↓
x + 2 = x² - 4x + 6
↓
x² - 5x + 4 = 0
↓
(x-4)(x-1) = 0
↓
x = 4 or x = 1
↓
(4,6) or (1,3)
```

E. Word Problems with Simultaneous Equations

Diagram 116: Problem-Solving Framework

```
WORD PROBLEM STRATEGY:
┌─────────────────────────────────┐
│ 1. READ problem carefully
│
│ 2. IDENTIFY what to find
│
│ 3. DEFINE variables
│
│ 4. TRANSLATE to equations
│
│ 5. SOLVE system
```



```
│ 6. INTERPRET solution
│
│ 7. CHECK answer makes sense
```

```
│
└─────────────────────────────────┘
```

COMMON WORD PROBLEM TYPES:

```
┌──────────────────────────┐
 1. Number Problems
│
 2. Age Problems

 3. Money/Cost Problems │
 4. Distance/Speed

 5. Mixture Problems

 6. Digit Problems

└──────────────────────────┘
```

TRANSLATION PHRASES:
"sum of" → +
"difference" → -
"total" → +
"more than" → +
"less than" → -
"twice" → 2×
"half" → ÷2 or ×1/2
"is/equals" → =

Diagram 117: Number Problem Example

PROBLEM: The sum of two numbers is 25
and their difference is 7.
Find the numbers.
STEP 1: Define variables
Let first number = x
Let second number = y
STEP 2: Translate to equations
"sum is 25"
x + y = 25 ... (1)
"difference is 7"
x - y = 7 ... (2)190
VISUAL REPRESENTATION:

```
├──x──┤
├─y── ┤
└──7──┘
├──25──┤
```

STEP 3: Choose method
Elimination (coefficients of y are equal)
STEP 4: Add equations
x + y = 25

x - y = 7
———————————
2x = 32
x = 16
ADDITION VISUAL:
x + y = 25
+ x - y = 7
—————————————
2x + 0 = 32
STEP 5: Find y
Substitute x = 16 into (1):
16 + y = 25
y = 9
SOLUTION:

First number = 16

Second number = 9

VERIFICATION:
Sum: 16 + 9 = 25 ✓
Difference: 16 - 9 = 7 ✓191
Diagram 118: Cost Problem Example
PROBLEM: At a shop, 3 pens and 2 books
cost ₦450. 5 pens and 4 books
cost ₦830. Find the cost of
one pen and one book.
STEP 1: Define variables
Let cost of 1 pen = ₦x
Let cost of 1 book = ₦y
VISUAL:
= ₦450
(3 pens) (2 books)
= ₦830
(5 pens)
(4 books)
STEP 2: Form equations
3x + 2y = 450 ... (1)
5x + 4y = 830 ... (2)
STEP 3: Use elimination
Multiply (1) by 2:
6x + 4y = 900 ... (3)
STEP 4: Subtract
(3) - (2):

$6x + 4y = 900$
$-(5x + 4y = 830)$

$x = 70$

STEP 5: Find y192

Substitute $x = 70$ into (1):

$3(70) + 2y = 450$
$210 + 2y = 450$
$2y = 240$
$y = 120$

SOLUTION:

Cost of 1 pen = ₦70

Cost of 1 book = ₦120

VERIFICATION:

Eq 1: $3(70) + 2(120) = 210 + 240 = 450$ ✓
Eq 2: $5(70) + 4(120) = 350 + 480 = 830$ ✓

COST TABLE:

Item	Qty	Price	Total
Pen	3	₦70	₦210
Book	2	₦120	₦240
₦450 ✓			
Pen	5	₦70	₦350
Book	4	₦120	₦480
₦830 ✓			

ANSWER: One pen costs ₦70
One book costs ₦120

Diagram 119: Age Problem Example

PROBLEM: A man is 3 times as old as his
son. In 12 years, he will be
twice as old as his son.
Find their present ages.
STEP 1: Define variables
Let son's present age = x years193
Let man's present age = y years
AGE TABLE:

Now
In 12 yrs

Son

x

x+12

Man

y

y+12

STEP 2: Form equations
"man is 3 times son's age"
y = 3x ... (1)
"in 12 years, twice as old"
y + 12 = 2(x + 12) ... (2)
VISUAL NOW:
Man's age: ●●●●●●●●● (3 times)
Son's age: ●●●
(1 times)
STEP 3: Simplify equation (2)
y + 12 = 2x + 24
y = 2x + 12 ... (3)
STEP 4: Use substitution
From (1): y = 3x
Substitute into (3):
3x = 2x + 12
x = 12
STEP 5: Find y
y = 3(12) = 36
SOLUTION:

Son's age = 12 years

```
│ Man's age = 36 years │
└────────────────────────┘
```
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```
VERIFICATION:
Now: Man is 36, Son is 12
36 = 3 × 12 ✓
In 12 years: Man is 48, Son is 24
48 = 2 × 24 ✓
TIMELINE:
Now
In 12 years
Son: 12 ──────────→ 24
↕×3
↕×2
Man: 36 ──────────→ 48
AGE RATIO:
Now: 36:12 = 3:1
Future: 48:24 = 2:1
ANSWER: Son is 12 years old
Man is 36 years old
```

Worked Examples with Complete Solutions:

Example 1: Solve by elimination: 3x + 4y = 25 2x - 3y = -6

Diagram 120: Complete Elimination Solution

```
Given: 3x + 4y = 25 ... (1)
2x - 3y = -6 ... (2)
STEP 1: Choose variable to eliminate
Let's eliminate x
STEP 2: Find LCM of x-coefficients
Coefficients: 3 and 2
LCM = 6
MULTIPLICATION STRATEGY:
┌─────────────────────────┐
│ Eq (1) × 2 → 6x + 8y
│
│ Eq (2) × 3 → 6x - 9y
│
└─────────────────────────┘
```
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```
STEP 3: Multiply equations
(1) × 2: 6x + 8y = 50 ... (3)
(2) × 3: 6x - 9y = -18 ... (4)
MULTIPLICATION CHECK:
(1) × 2: 3x×2 + 4y×2 = 25×2
6x + 8y = 50 ✓
(2) × 3: 2x×3 - 3y×3 = -6×3
6x - 9y = -18 ✓
STEP 4: Subtract equations
```

```
(3) - (4):
6x + 8y = 50
-(6x - 9y = -18)
──────────────────
0 + 17y = 68
SUBTRACTION BREAKDOWN:
┌───────────────────────────┐
 6x - 6x = 0

 8y - (-9y) = 8y+9y = 17y │
 50 - (-18) = 50+18 = 68 │
└───────────────────────────┘
STEP 5: Solve for y
17y = 68
y = 4
STEP 6: Substitute into original (1)
3x + 4(4) = 25
3x + 16 = 25
3x = 9
x = 3
SOLUTION BOX:
┌───────────────┐
 x = 3

 y = 4
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└───────────────┘
VERIFICATION:
Equation 1: 3(3) + 4(4) = 9 + 16 = 25 ✓
Equation 2: 2(3) - 3(4) = 6 - 12 = -6 ✓
COMPLETE SOLUTION PATH:
3x + 4y = 25
2x - 3y = -6
↓ (×2, ×3)
6x + 8y = 50
6x - 9y = -18
↓ (subtract)
17y = 68
↓ (÷17)
y = 4
↓ (substitute)
3x + 16 = 25
↓
x = 3
GRAPHICAL REPRESENTATION:
```

```
Lines intersect at (3, 4)
ANSWER: x = 3, y = 4
```

Example 2: Solve by substitution: y = 3x - 5 2x + 3y = 4

Diagram 121: Complete Substitution Solution

```
Given: y = 3x - 5 ... (1)
2x + 3y = 4 ... (2)197
STEP 1: Identify substitution
Equation (1) already solved for y!
STEP 2: Substitute into equation (2)
2x + 3(3x - 5) = 4
SUBSTITUTION DIAGRAM:
2x + 3y = 4
↓
2x + 3(3x-5) = 4
└───────┘
Replace y with 3x-5
STEP 3: Expand brackets
2x + 9x - 15 = 4
EXPANSION VISUAL:
3(3x - 5)
= 3×3x + 3 × (-5)
= 9x - 15
STEP 4: Simplify
11x - 15 = 4
11x = 19
x = 19/11
STEP 5: Find y using equation (1)
y = 3(19/11) - 5
y = 57/11 - 55/11
y = 2/11
EXACT SOLUTION:
┌───────────────────┐
  x = 19/11

  y = 2/11

└───────────────────┘
DECIMAL APPROXIMATION:
┌───────────────────┐
  x ≈ 1.73

  y ≈ 0.18

└───────────────────┘VERIFICATION:
Eq 1: y = 3(19/11) - 5
= 57/11 - 55/11
```

= 2/11 ✓
Eq 2: 2(19/11) + 3(2/11)
= 38/11 + 6/11
= 44/11
= 4 ✓
SOLUTION FLOWCHART:
y = 3x - 5
2x + 3y = 4
↓
2x + 3(3x-5) = 4
↓
2x + 9x - 15 = 4
↓
11x = 19
↓
x = 19/11
↓
y = 3(19/11) - 5
↓
y = 2/11
FRACTION WORK:
For y:
3(19/11) - 5
= 57/11 - 5
= 57/11 - 55/11 (LCD = 11)
= (57-55)/11
= 2/11
ANSWER: x = 19/11, y = 2/11
or x ≈ 1.73, y ≈ 0.18

Example 3: Solve: $y = x^2 - 2x + 3$ $y = 2x + 1$

Diagram 122: Linear-Quadratic Solution

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Given: $y = x^2 - 2x + 3$ ... (1) [Quadratic]
y = 2x + 1
... (2) [Linear]
STEP 1: Equate the expressions
Since both equal y:
$x^2 - 2x + 3 = 2x + 1$
EQUATION VISUAL:
$y = x^2 - 2x + 3$
y = 2x + 1
↓ (set equal)
$x^2 - 2x + 3 = 2x + 1$
STEP 2: Rearrange to standard form
$x^2 - 2x + 3 - 2x - 1 = 0$

$x^2 - 4x + 2 = 0$

REARRANGEMENT STEPS:

$x^2 - 2x + 3 = 2x + 1$

$x^2 - 2x - 2x = 1 - 3$

$x^2 - 4x = -2$

$x^2 - 4x + 2 = 0$

STEP 3: Use quadratic formula

$a = 1,\ b = -4,\ c = 2$

$x = (-b \pm \sqrt{(b^2 - 4ac)})/2a$

$x = (4 \pm \sqrt{(16 - 8)})/2$

$x = (4 \pm \sqrt{8})/2$

$x = (4 \pm 2\sqrt{2})/2$

$x = 2 \pm \sqrt{2}$

DISCRIMINANT:

$\Delta = (-4)^2 - 4(1)(2)$

$= 16 - 8$

$= 8 > 0$

Two distinct solutions ✓

STEP 4: Calculate x values200

$x_1 = 2 + \sqrt{2} \approx 2 + 1.414 = 3.414$

$x_2 = 2 - \sqrt{2} \approx 2 - 1.414 = 0.586$

STEP 5: Find corresponding y values

For $x_1 = 2 + \sqrt{2}$:

$y_1 = 2(2 + \sqrt{2}) + 1$

$= 4 + 2\sqrt{2} + 1$

$= 5 + 2\sqrt{2}$

≈ 7.83

For $x_2 = 2 - \sqrt{2}$:

$y_2 = 2(2 - \sqrt{2}) + 1$

$= 4 - 2\sqrt{2} + 1$

$= 5 - 2\sqrt{2}$

≈ 2.17

EXACT SOLUTIONS:

Solution 1: $(2+\sqrt{2},\ 5+2\sqrt{2})$

Solution 2: $(2-\sqrt{2},\ 5-2\sqrt{2})$

APPROXIMATE SOLUTIONS:

Solution 1: $(3.41,\ 7.83)$

Solution 2: $(0.59,\ 2.17)$

VERIFICATION (using x ≈ 3.41):
Eq 1: $y = (3.41)^2 - 2(3.41) + 3$
≈ 11.63 - 6.82 + 3
≈ 7.81 ≈ 7.83 ✓
Eq 2: y = 2(3.41) + 1
= 6.82 + 1
= 7.82 ≈ 7.83 ✓201
0 1 2 3 4
Parabola: $y = x^2 - 2x + 3$
Line: y = 2x + 1
Intersections at both points
SOLUTION PATH:
$y = x^2 - 2x + 3$
y = 2x + 1
↓
$x^2 - 2x + 3 = 2x + 1$
↓
$x^2 - 4x + 2 = 0$
↓
$x = (4 \pm \sqrt{8})/2$
↓
$x = 2 \pm \sqrt{2}$
↓
$(2+\sqrt{2}, 5+2\sqrt{2})$ or $(2-\sqrt{2}, 5-2\sqrt{2})$

Example 4: At a fruit stall, 5 oranges and 3 apples cost ₦200. 7 oranges and 5 apples cost ₦310. Find the cost of each fruit.

Diagram 123: Cost Problem Complete Solution

Given Information:
• 5 oranges + 3 apples = ₦200
• 7 oranges + 5 apples = ₦310
VISUAL REPRESENTATION:
Scenario 1:202
????? ??? = ₦200
Scenario 2:
??????? ????? = ₦310
STEP 1: Define variables
Let cost of 1 orange = ₦x
Let cost of 1 apple = ₦y
STEP 2: Form equations
5x + 3y = 200 ... (1)
7x + 5y = 310 ... (2)
STEP 3: Use elimination method
Choose to eliminate x
LCM of 5 and 7 = 35

MULTIPLICATION PLAN:

Eq (1) × 7 → 35x + 21y

Eq (2) × 5 → 35x + 25y

STEP 4: Multiply equations
(1) × 7: 35x + 21y = 1400 ... (3)
(2) × 5: 35x + 25y = 1550 ... (4)
STEP 5: Subtract
(4) - (3):
35x + 25y = 1550
-(35x + 21y = 1400)
———————————————

0 + 4y = 150
SUBTRACTION DETAIL:

35x - 35x = 0

25y - 21y = 4y

1550 - 1400 = 150

STEP 6: Solve for y
4y = 150
y = 37.5
STEP 7: Substitute into (1)
5x + 3(37.5) = 200
5x + 112.5 = 200
5x = 87.5
x = 17.5
SOLUTION:

Cost of 1 orange = ₦17.50 |
Cost of 1 apple = ₦37.50

VERIFICATION:
Scenario 1:
5(17.5) + 3(37.5)
= 87.5 + 112.5
= 200 ✓
Scenario 2:

7(17.5) + 5(37.5)
= 122.5 + 187.5
= 310 ✓
COST BREAKDOWN TABLE:

Fruit	Qty	Price	Total
Orange	5	₦17.50	₦87.50
Apple	3	₦37.50	₦112.50
₦200 ✓			
Orange	7	₦17.50	₦122.50
Apple	5	₦37.50	₦187.50
₦310 ✓			

PRICE COMPARISON:
Apple is more expensive:
₦37.50 - ₦17.50 = ₦20 more
Ratio: 37.5:17.5 ≈ 2.14:1
(Apples cost about twice as much)
ANSWER:
One orange costs ₦17.50
One apple costs ₦37.50

Class Activity:

Activity 1: Method Comparison

Diagram 124: Solve Using Both Methods

Solve using BOTH elimination and substitution:
x + y = 10
x - y = 2
METHOD 1: ELIMINATION
x + y = 10
+ x - y = 2

2x = 12
x = 6
Then: y = 4
METHOD 2: SUBSTITUTION
From (1): y = 10 - x
Into (2): x - (10-x) = 2
2x = 12

```
x = 6
Then: y = 4
Same answer! ✓
```

Activity 2: Real-World Scenarios

204In groups, create word problems involving:

• Shopping (costs of items)
• Geometry (perimeter and area)
• Age relationships
• Mixture problems

Activity 3: Graphical Verification

Plot these systems on graph paper:

1. $y = x + 1$ and $y = 3 - x$
2. $2x + y = 6$ and $x - y = 3$

Find solutions graphically, then verify algebraically.

Evaluation Questions:

1. Solve by elimination: $2x + 3y = 12$ $3x - 2y = 5$
2. Solve by substitution: $y = 2x - 1$ $3x + y = 14$
3. Solve the system: $y = x^2 + 1$ $y = 2x + 3$
4. Three books and two pens cost ₦350. Five books and three pens cost ₦550. Find the cost of each.
5. The sum of two numbers is 50. If the larger number is 10 more than the smaller, find both numbers.

Assignment:

1. Solve by elimination: (a) $3x + 2y = 16$, $5x - 3y = 2$ (b) $4x + 5y = 23$, $3x + 4y = 18$
2. Solve by substitution: (a) $x = 3y - 2$, $2x + y = 11$ (b) $y = 2x + 1$, $3x - 4y = -13$
3. Solve: (a) $y = x^2 - 4x + 5$, $y = 2x - 1$ (b) $y = x^2 + 2x + 1$, $y = x + 3$
4. At a restaurant, 4 meals and 3 drinks cost ₦2,400. 6 meals and 5 drinks cost ₦3,700. Find the cost of one
meal and one drink.
5. The perimeter of a rectangle is 40 cm. The length is 4 cm more than the width. Find the dimensions.

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WEEK 9: GRAPHICAL SOLUTIONS

Learning Objectives:

By the end of the lesson, students should be able to:

1. Draw graphs of linear equations
2. Draw graphs of quadratic equations (parabolas)
3. Interpret graphs of equations
4. Solve simultaneous equations graphically
5. Find solutions from intersection points
6. Compare graphical and algebraic solutions
7. Apply graphical methods to real-life problems

Entry Behaviour:

Students can plot points on coordinate axes, understand linear and quadratic equations, and solve equations algebraically.

Instructional Materials:

- Graph paper
- Rulers and pencils
- Set squares
- Colored pencils/markers
- Coordinate grid charts
- Calculator
- Transparencies for overlays

Content:

A. Coordinate System Review

Diagram 125: The Cartesian Plane

```
THE COORDINATE SYSTEM
y-axis (vertical)
↓
QUADRANTS:
┌─────────────────────────────┐
│ I:
x > 0, y > 0 (+,+)
│
│ II: x < 0, y > 0 (-,+)
│
│ III: x < 0, y < 0 (-,-)
│
│ IV: x > 0, y < 0 (+,-)
│
└─────────────────────────────┘
PLOTTING POINTS:
Point (3, 2):
• Start at origin (0, 0)
• Move 3 units right (x = 3)
• Move 2 units up (y = 2)
• Mark the point
VISUAL:
y
3 │
2 │
● (3,2)
1 │
├──────┼──────→ x
0 1 2 3 4
KEY FEATURES:
• Origin: (0, 0)
• x-intercept: where line crosses x-axis (y = 0)
• y-intercept: where line crosses y-axis (x = 0)
SCALE:
```

Important to choose appropriate scale!
Small values: 1 square = 1 unit
Large values: 1 square = 2, 5, or 10 units
EXAMPLE SCALES:208
1 cm = 1 unit (for values 0-20)
1 cm = 2 units (for values 0-40)
1 cm = 5 units (for values 0-100)

B. Graphing Linear Equations

Diagram 126: Methods for Drawing Linear Graphs

LINEAR EQUATION FORM:
y = mx + c
Where:
m = gradient (slope)
c = y-intercept
METHOD 1: TABLE OF VALUES

1. Choose x values
2. Calculate y values
3. Create table
4. Plot points
5. Draw straight line

EXAMPLE: y = 2x + 1
TABLE:

x	-2	-1	0	1	2
y	-3	-1	1	3	5

Calculations:
x = -2: y = 2(-2) + 1 = -3
x = -1: y = 2(-1) + 1 = -1
x = 0: y = 2(0) + 1 = 1
x = 1: y = 2(1) + 1 = 3
x = 2: y = 2(2) + 1 = 5

METHOD 2: INTERCEPT METHOD
(Faster for equations in form ax + by = c)
Steps:
1. Find y-intercept (set x = 0)
2. Find x-intercept (set y = 0)

3. Plot both points
4. Draw line through them

EXAMPLE: 2x + 3y = 6

y-intercept (x = 0):
3y = 6
y = 2
Point: (0, 2)

x-intercept (y = 0):
2x = 6
x = 3
Point: (3, 0)

METHOD 3: GRADIENT-INTERCEPT
(For equations in form y = mx + c)

Steps:
1. Plot y-intercept (0, c)
2. Use gradient m = rise/run
3. From y-intercept, count:
• run (horizontal)
• rise (vertical)
4. Mark second point
5. Draw line

EXAMPLE: y = (3/2)x - 1

y-intercept: c = -1
Point: (0, -1)

Gradient: m = 3/2 = rise/run

From (0, -1):
• Run 2 right
• Rise 3 up
New point: (2, 2)

GRADIENT VISUAL:

```
● (2, 2)
/│
/ │ rise = 3
/ │
●───┘ (0, -1)
run = 2211
```

CHARACTERISTICS:

```
┌──────────────────────────┐
│ Positive gradient /
│
│ (line slopes upward)
│
│
│
│
│ Negative gradient \
```

```
│
│ (line slopes downward)
│
│
│ Zero gradient ──
│
│ (horizontal line)
│
│
│ Undefined gradient │
│
│ (vertical line)
│
└───────────────────────────┘
```

Diagram 127: Special Linear Graphs

```
HORIZONTAL LINES:
Form: y = k (constant)
Example: y = 3
y
4 │
3 ●═══════════
2 │
1 │
└──────────→ x
All points have y = 3
Gradient = 0
```

```
VERTICAL LINES:
Form: x = k (constant)212
Example: x = 2
y
4 │
3 │ ║
2 │ ║
1 │ ║
└──║──────→ x
2
All points have x = 2
Gradient = undefined
```

```
LINES THROUGH ORIGIN:
Form: y = mx (no constant)
Example: y = 2x
```

Passes through (0, 0)

PARALLEL LINES:
Same gradient, different intercepts
$y = 2x + 3$
$y = 2x - 1213$
 ($y = 2x - 1$)
Both have $m = 2$
Never intersect!

PERPENDICULAR LINES:
Gradients multiply to give -1
If $m_1 = 2$, then $m_2 = -1/2$

$m_1 \times m_2 = 2 \times (-1/2) = -1$

C. Graphing Quadratic Equations

Diagram 128: Parabola Basics

QUADRATIC FORM:
$y = ax^2 + bx + c$
PARABOLA SHAPE:

```
┌──────────────────────────────┐
│ a > 0: Opens upward ∪
│
│ a < 0: Opens downward ∩
│
└──────────────────────────────┘
```

UPWARD ($a > 0$):
Minimum point at vertex
DOWNWARD ($a < 0$):
Maximum point at vertex
KEY FEATURES:

- Vertex (turning point)
- Axis of symmetry
- y-intercept (c)
- x-intercepts (roots)
- Domain: all real x
- Range: depends on vertex

VERTEX FORMULA:

x-coordinate of vertex = -b/(2a)
Then substitute to find y
AXIS OF SYMMETRY:
Vertical line through vertex
x = -b/(2a)
axis

Diagram 129: Drawing Parabolas - Step by Step

EXAMPLE: Draw $y = x^2 - 4x + 3$
STEP 1: Create table of values
Choose x values around vertex
x-coordinate of vertex:
$x = -b/(2a) = -(-4)/(2\times1) = 2$
Choose x from 0 to 4:
TABLE:

x	0	1	2	3	4
y	3	0	-1	0	3

CALCULATIONS:
x = 0: y = 0 - 0 + 3 = 3
x = 1: y = 1 - 4 + 3 = 0
x = 2: y = 4 - 8 + 3 = -1
x = 3: y = 9 - 12 + 3 = 0
x = 4: y = 16 - 16 + 3 = 3216
STEP 2: Plot points
●
Points: (0,3), (1,0), (2,-1), (3,0), (4,3)
STEP 3: Draw smooth curve
0 1 2 3 4
FEATURES:
• Vertex: (2, -1) ← minimum
• Axis of symmetry: x = 2
• y-intercept: (0, 3)
• x-intercepts: (1, 0) and (3, 0)
[These are the roots!]
SYMMETRY CHECK:
Points equidistant from x = 2
have same y-value:
(0, 3) and (4, 3) ✓
(1, 0) and (3, 0) ✓

TIPS FOR ACCURATE DRAWING:
✓ Use at least 5-7 points
✓ Include vertex

✓ Mark intercepts clearly✓ Use smooth curve (not straight lines)
✓ Check symmetry
✓ Extend curve slightly beyond table values
✓ Use pencil for corrections
COMMON MISTAKES:
✗ Connecting points with straight lines
✗ Not enough points
✗ Asymmetric curve
✗ Wrong shape (opening direction)

D. Solving Simultaneous Equations Graphically

Diagram 130: Graphical Solution Method

PRINCIPLE:
Solution = Intersection point(s)
TWO LINEAR EQUATIONS:
Usually ONE intersection point
EXAMPLE:
y = 2x + 1 ... (1)
y = -x + 4 ... (2)
STEP 1: Draw both lines
For (1): y = 2x + 1

x	0	1	2
y	1	3	5

For (2): y = -x + 4

x	0	1	2
y	4	3	2

217218
STEP 2: Plot on same axes

```
y
5 |
● Line 1
4 ●────●────
3 | \ X /
2 | ●───● Line 2
1 ● \
0 └───────────→ x
0 1 2 3
```

STEP 3: Read intersection point

```
●───X─── Intersection at (1, 3)
↑
```

Solution: x = 1, y = 3
VERIFICATION:
Eq 1: y = 2(1) + 1 = 3 ✓
Eq 2: y = -(1) + 4 = 3 ✓

LINEAR + QUADRATIC:
Can have 0, 1, or 2 solutions
EXAMPLE:
$y = x^2 - 2x - 3$ (parabola)
y = x + 1
(line)
-4 │ V
Two intersections:
Solution 1: (-1, 0)
Solution 2: (4, 5)219
VERIFICATION:
For (-1, 0):
Parabola: 1 + 2 - 3 = 0 ✓
Line: -1 + 1 = 0 ✓
For (4, 5):
Parabola: 16 - 8 - 3 = 5 ✓
Line: 4 + 1 = 5 ✓

READING SOLUTIONS:
ACCURATE READING:
✓ Use sharp pencil
✓ Read to nearest 0.1 or 0.5
✓ Check scale carefully
✓ Verify algebraically
GRID INTERSECTION:
If intersection falls on grid point:
Solution is exact
If between grid points:
Solution is approximate
EXAMPLE:
│
──X── Intersection at (2.5, 3)
│
2
3
Approximate: x = 2.5, y = 3

Diagram 131: Special Cases

CASE 1: PARALLEL LINES
No intersection = No solution
│ Line 1 (y = 2x + 3)

```
/ Line 2 (y = 2x - 1)
Same gradient, different intercepts
Lines never meet!
─────────────────────────────────
CASE 2: COINCIDENT LINES
Same line = Infinite solutions
y
│ /
│/ Both lines overlap
●═══════════→ x
Every point on line is a solution!
─────────────────────────────────
CASE 3: LINE MISSES PARABOLA
No intersection = No real solutions
y
│ ___
│ /
\ Parabola
|/
\
●───────────→ x
Line below
Line and parabola don't meet
(System has complex solutions)
─────────────────────────────────
CASE 4: LINE TANGENT TO PARABOLA221
One touch point = One solution
Line touches parabola at exactly
one point (tangent)
```

E. Interpreting Graphs

Diagram 132: Reading Information from Graphs

```
WHAT GRAPHS TELL US:
1. INTERCEPTS
┌─────────────────────────────┐
│ y-intercept: where x = 0
│
│ x-intercept: where y = 0
│
└─────────────────────────────┘
x-intercept (2, 0)
─────────────────────────────────
2. GRADIENT (SLOPE)
┌─────────────────────────────┐
│ Steeper line = larger m
│
│ Positive m: /
```

Negative m: \

Zero m: ──

CALCULATING FROM GRAPH:
Pick two points: (x_1, y_1) and (x_2, y_2)
$m = (y_2 - y_1)/(x_2 - x_1)$
= rise/run
EXAMPLE:
Points: (0, 1) and (2, 5)
$m = (5 - 1)/(2 - 0)$
$= 4/2$
$= 2$

---

3. ROOTS/SOLUTIONS

Where graph crosses x-axis
These are solutions to

equation when $y = 0$

Roots: $x = -1$ and $x = 3$
(Solutions to $x^2 - 2x - 3 = 0$)

---

4. VERTEX/TURNING POINT

Maximum or minimum point

Changes direction

vertex
Read coordinates: (2, -1)
This is minimum point

---

5. RANGE OF VALUES

For what x is y positive?
For what x is $y > 5$?

y
6 │─●── y = 5 line

From graph:
• y > 0 when -2 < x < 4
• y > 5 when x is between
the points where y = 5

---

6. RATE OF CHANGE

```
┌─────────────────────────────┐
│ How fast y changes with x │
│ = gradient at that point
│
└─────────────────────────────┘
```

For linear: constant rate224
For curved: varying rate
EXAMPLE: Distance-Time Graph

```
Distance
│
/
│
/ Steeper = faster
│ /
│ /
└────────→ Time
```

Gradient = speed

Worked Examples with Complete Solutions:

Example 1: Draw the graph of $y = 3x - 2$ for $-1 \leq x \leq 3$

Diagram 133: Complete Linear Graph Solution

Given: y = 3x - 2
Domain: -1 ≤ x ≤ 3
STEP 1: Create table of values
Choose x: -1, 0, 1, 2, 3
CALCULATIONS:
x = -1: y = 3(-1) - 2 = -3 - 2 = -5
x = 0: y = 3(0) - 2 = 0 - 2 = -2
x = 1: y = 3(1) - 2 = 3 - 2 = 1
x = 2: y = 3(2) - 2 = 6 - 2 = 4
x = 3: y = 3(3) - 2 = 9 - 2 = 7
TABLE:

x	-1	0	1	2	3	
y 7	-5	-2	1	4		

STEP 2: Choose scale
x-axis: 1 cm = 1 unit
y-axis: 1 cm = 1 unit225

STEP 3: Plot points

-1 0 1 2 3

STEP 4: Draw straight line

0 ●—/———————→ x

-1 0 1 2 3

FEATURES:

• Gradient: m = 3 (steep upward slope)

• y-intercept: (0, -2)• x-intercept: Set y = 0

0 = 3x - 2

3x = 2

x = 2/3 ≈ 0.67

Point: (0.67, 0)

GRADIENT CHECK:

Using (0, -2) and (1, 1):

m = (1 - (-2))/(1 - 0)

= 3/1

= 3 ✓

EQUATION VERIFICATION:

For x = 2:

y = 3(2) - 2 = 4

Point (2, 4) is on line ✓

DOMAIN AND RANGE:

Domain: -1 ≤ x ≤ 3

Range: -5 ≤ y ≤ 7

Example 2: Draw the graph of $y = x^2 - 4x + 3$ for $0 \leq x \leq 4$

Diagram 134: Complete Parabola Solution

Given: $y = x^2 - 4x + 3$

Domain: 0 ≤ x ≤ 4

STEP 1: Find vertex

x-coordinate: x = -b/(2a)

= -(-4)/(2×1)

= 4/2

= 2

y-coordinate: $y = (2)^2 - 4(2) + 3$

= 4 - 8 + 3

= -1

Vertex: (2, -1)

STEP 2: Create table

226227

Include vertex and points on each side

x	0	1	2	3	4
y	3	0	-1	0	3

CALCULATIONS:
$x = 0$: $y = 0 - 0 + 3 = 3$
$x = 1$: $y = 1 - 4 + 3 = 0$
$x = 2$: $y = 4 - 8 + 3 = -1$
$x = 3$: $y = 9 - 12 + 3 = 0$
$x = 4$: $y = 16 - 16 + 3 = 3$
STEP 3: Plot points
●
0 1 2 3 4
STEP 4: Draw smooth curve
-2 |
(2,-1)
0 1 2 3 4
KEY FEATURES:

- Opens upward ($a = 1 > 0$)
- Vertex: (2, -1) minimum 228
- Axis of symmetry: $x = 2$
- y-intercept: (0, 3)
- x-intercepts: (1, 0), (3, 0)
- Domain: $0 \le x \le 4$
- Range: $y \ge -1$

SYMMETRY CHECK:
Points equidistant from $x = 2$:
- (0, 3) and (4, 3) ✓
- (1, 0) and (3, 0) ✓

Parabola is symmetric!
ROOTS:
From graph: $x = 1$ and $x = 3$
These are solutions to:
$x^2 - 4x + 3 = 0$
Verify by factoring:
$(x - 1)(x - 3) = 0$
$x = 1$ or $x = 3$ ✓
ANALYSIS:
- Minimum value: $y = -1$ at $x = 2$
- $y > 0$ when $x < 1$ or $x > 3$
- $y < 0$ when $1 < x < 3$
- $y = 0$ when $x = 1$ or $x = 3$

Example 3: Solve graphically: $y = 2x + 1$ $y = -x + 7$

Diagram 135: Graphical Solution of Linear System

Given:

y = 2x + 1 ... (1)

y = -x + 7 ... (2)

STEP 1: Table for equation (1)

y = 2x + 1229

x	0	1	2	3	
y	1	3	5	7	

Calculations:

x = 0: y = 0 + 1 = 1

x = 1: y = 2 + 1 = 3

x = 2: y = 4 + 1 = 5

x = 3: y = 6 + 1 = 7

STEP 2: Table for equation (2)

y = -x + 7

x 6	0	2	4		
y	7	5	3	1	

Calculations:

x = 0: y = 0 + 7 = 7

x = 2: y = -2 + 7 = 5

x = 4: y = -4 + 7 = 3

x = 6: y = -6 + 7 = 1

STEP 3: Plot both lines

0 1 2 3 4 5230

STEP 4: Identify intersection

0 1 2 3 4

READING SOLUTION:

Intersection at (2, 5)

Therefore:

$$\boxed{\begin{array}{l} x = 2 \\ y = 5 \end{array}}$$

STEP 5: Verify algebraically

Equation 1: y = 2(2) + 1 = 5 ✓

Equation 2: y = -(2) + 7 = 5 ✓

ALTERNATIVE VERIFICATION:
Substitute into originals:
Point (2, 5):

- 5 = 2(2) + 1 → 5 = 5 ✓
- 5 = -2 + 7 → 5 = 5 ✓

GRADIENT ANALYSIS:
Line 1: m = 2 (positive, upward)
Line 2: m = -1 (negative, downward)
Different gradients → one intersection ✓
ANSWER: Solution is x = 2, y = 5
or point (2, 5)231
COMPARISON WITH ALGEBRA:
Substitution method:
2x + 1 = -x + 7
3x = 6
x = 2, y = 5 ✓
Same answer!

Example 4: Solve graphically: $y = x^2 - x - 2$ $y = x + 1$

Diagram 136: Linear-Quadratic Graphical Solution

Given:
$y = x^2 - x - 2$ ... (1) Parabola
$y = x + 1$
... (2) Line
STEP 1: Table for parabola
$y = x^2 - x - 2$

x	-2	-1	0	1	2	3
y	4	0	-2	-2	0	4

Calculations:
x = -2: y = 4 + 2 - 2 = 4
x = -1: y = 1 + 1 - 2 = 0
x = 0: y = 0 - 0 - 2 = -2
x = 1: y = 1 - 1 - 2 = -2
x = 2: y = 4 - 2 - 2 = 0
x = 3: y = 9 - 3 - 2 = 4
STEP 2: Table for line
$y = x + 1$

x	-2	-1	0	1	2	3
y	-1	0	1	2	3	4

```
│
└────┴────┴────┴────┴────┴────┴────┘
```
232

```
STEP 3: Plot both graphs
-2-1 0 1 2 3
STEP 4: Mark intersections
-2-1 0 1 2 3
READING SOLUTIONS:
Intersection 1: (-1, 0)
Intersection 2: (3, 4)
Therefore:
┌─────────────────────────┐
│ Solution 1: x=-1, y=0 │
│ Solution 2: x=3, y=4
│
└─────────────────────────┘
VERIFICATION:
Solution 1: (-1, 0)
Parabola: (-1)² - (-1) - 2 = 1 + 1 - 2 = 0 ✓
Line: -1 + 1 = 0 ✓Solution 2: (3, 4)
Parabola: 3² - 3 - 2 = 9 - 3 - 2 = 4 ✓
Line: 3 + 1 = 4 ✓
ALGEBRAIC CHECK:
Set equations equal:
x² - x - 2 = x + 1
x² - 2x - 3 = 0
(x - 3) (x + 1) = 0
x = 3 or x = -1 ✓
When x = -1: y = 0
When x = 3: y = 4
Matches graphical solution!
ANALYSIS:
• Two distinct intersections
• Parabola opens upward
• Line has positive gradient
• Solutions are exact (on grid points)
ANSWER:
x = -1, y = 0 OR x = 3, y = 4
Points: (-1, 0) and (3, 4)
```

Example 5: A taxi charges a fixed fee of ₦200 plus ₦50 per km. A second taxi charges ₦100 plus ₦75 per km. Draw graphs and find when costs are equal.

Diagram 137: Real-World Application

```
Problem Setup:
Taxi A: Cost = 200 + 50d
Taxi B: Cost = 100 + 75d
(where d = distance in km)
```

STEP 1: Create equations
Let C = Cost, d = distance
Taxi A: C = 50d + 200 ... (1)
Taxi B: C = 75d + 100 ... (2)
STEP 2: Table for Taxi A
233234
C = 50d + 200

d	0	2	4	6	8
C	200	300	400	500	600

STEP 3: Table for Taxi B
C = 75d + 100

d	0	2	4	6	8
C	100	250	400	550	700

STEP 4: Plot graphs
0└──────────→ d (km)
0 2 4 6 8
STEP 5: Find intersection
0 2 4 6 8235
READING SOLUTION:
Intersection at (4, 400)
Distance: 4 km
Cost: ₦400
INTERPRETATION:

When distance = 4 km

Both taxis cost ₦400

For d < 4 km: Taxi B cheaper

For d > 4 km: Taxi A cheaper

For d = 4 km: Same cost

VERIFICATION:

Taxi A at 4 km: $50(4) + 200 = 400$ ✓
Taxi B at 4 km: $75(4) + 100 = 400$ ✓
ALGEBRAIC SOLUTION:
$50d + 200 = 75d + 100$
$200 - 100 = 75d - 50d$
$100 = 25d$
$d = 4$ km ✓
DECISION TABLE:

d	Taxi A	Taxi B	Cheaper
2 km	₦300	₦250	B
4 km	₦400	₦400	Same
6 km	₦500	₦550	A

PRACTICAL ADVICE:
• Short trips (< 4 km): Choose Taxi B
• Long trips (> 4 km): Choose Taxi A
• Exactly 4 km: Either taxi ANSWER:
Costs are equal at 4 km (₦400)

Class Activity:

Activity 1: Graphing Competition

Diagram 138: Timed Exercise

In pairs, accurately draw:
1. $y = 2x - 3$ (use 3 colors for:
• table, • points, • line)
2. $y = x^2 - 4$ (mark all key features)
3. Solve graphically:
$y = x + 2$
$y = 5 - x$

Award points for:
✓ Accurate plotting
✓ Clear labeling
✓ Correct solution

✓ Neat presentation

Activity 2: Real-World Modeling

Create and graph a real-world scenario:

- Phone plan comparison
- Savings plan growth
- Temperature change over time
- Distance-time for journey

Activity 3: Error Detection

Find errors in provided graphs:

- Wrong scale
- Incorrect points
- Curved line instead of straight

236• Missing labels

Evaluation Questions:

1. Draw the graph of $y = 3x + 2$ for $-2 \leqslant x \leqslant 2$.
2. Draw $y = x^2 - 3x + 2$ for $0 \leqslant x \leqslant 3$. State the minimum point.
3. Solve graphically: $y = 2x - 1$ $y = -x + 5$
4. From the graph of $y = x^2 - 4x + 3$, find: (a) The roots (b) The vertex (c) The y-intercept
5. A company charges ₦500 setup fee plus ₦100 per hour. Another charges ₦300 plus ₦150 per hour. Graph both and find when costs are equal.

Assignment:

1. Draw graphs for $-3 \leqslant x \leqslant 3$: (a) $y = x - 2$ (b) $y = -2x + 1$ (c) $y = (1/2)x + 3$
2. Draw $y = x^2 + 2x - 3$ for $-4 \leqslant x \leqslant 2$. Find: (a) Vertex (b) Axis of symmetry (c) Roots (d) Range
3. Solve graphically and verify algebraically: (a) $y = x + 1$ and $y = 4 - x$ (b) $y = 2x - 3$ and $y = -x + 6$
4. Solve graphically: $y = x^2$ $y = 2x + 3$
5. A phone plan costs ₦1000 monthly plus ₦5 per minute. Another costs ₦500 plus ₦10 per minute. (a) Write equations for both plans (b) Draw graphs for 0 to 200 minutes (c) Find when costs are equal (d) Which is better for 150 minutes?

Diagram 140: Essential Formulas Summary

KEY FORMULAS TO REMEMBER:

STANDARD FORM

$A \times 10^n$ (where $1 \leq A < 10$)

240

PERCENTAGE ERROR

% Error = |Approx-Exact| × 100

Exact

LOGARITHMS

$\log(M \times N) = \log M + \log N$

$\log(M \div N) = \log M - \log N$

$\log(M^n) = n \log M$

ARITHMETIC PROGRESSION

$T_n = a + (n-1)d$

$S_n = n/2[2a + (n-1)d]$

or $S_n = n/2(a + l)$

QUADRATIC EQUATIONS

$x = -b \pm \sqrt{(b^2-4ac)}$

2a

Sum of roots: $\alpha+\beta = -b/a$

Product: $\alpha\beta = c/a$

Discriminant: $\Delta = b^2-4ac$

B. Comprehensive Revision Questions

Mixed Topic Problems:

Problem 1: Combining Multiple Topics

Diagram 141: Multi-Concept Problem

QUESTION:
The roots of the equation $x^2 - px + 12 = 0$are in arithmetic progression with common
difference 1. Find p and solve the equation.
SOLUTION:
STEP 1: Set up using AP concept
Let roots be $(a-d)$ and $(a+d)$
Given: common difference = 1, so $d = 1$
Roots: $(a-1)$ and $(a+1)$
STEP 2: Use product of roots
Product = $c/a = 12/1 = 12$
$(a-1)(a+1) = 12$
$a^2 - 1 = 12$
$a^2 = 13$
$a = \sqrt{13}$ (taking positive)
STEP 3: Find p using sum
Sum = $-b/a = p/1 = p$
$(a-1) + (a+1) = p$
$2a = p$
$p = 2\sqrt{13}$
STEP 4: Write equation
$x^2 - 2\sqrt{13}x + 12 = 0$
STEP 5: Roots are
$x = \sqrt{13} \pm 1$
VERIFICATION:
Sum: $(\sqrt{13}-1) + (\sqrt{13}+1) = 2\sqrt{13}$ ✓
Product: $(\sqrt{13}-1)(\sqrt{13}+1) = 13-1 = 12$ ✓

Topics Used:
✓ Quadratic equations
✓ Sum and product of roots
✓ Arithmetic progression
✓ Surds

Problem 2: Real-World Application

241**Diagram 142: Practical Problem**

QUESTION:
A number in standard form is 3.5×10^4.
If it's increased by 20%, express the
new number in standard form correct to
3 significant figures.
SOLUTION:
STEP 1: Find actual number
$3.5 \times 10^4 = 35{,}000$
STEP 2: Calculate 20% increase
20% of 35,000 = 0.2 × 35,000
= 7,000
New number = 35,000 + 7,000
= 42,000
STEP 3: Express in standard form
$42{,}000 = 4.2 \times 10^4$
STEP 4: Check significant figures
4.2 already has 2 s.f.
To 3 s.f.: 4.20×10^4
ANSWER: 4.20×10^4
Topics Used:
✓ Standard form
✓ Percentages
✓ Significant figures

Problem 3: Sequential Problem

Diagram 143: Multi-Step Question

QUESTION:
Find the 10th term of the AP: 3, 7, 11, ...
Then use this as the constant term to
242form a quadratic equation with roots
2 and 5. Solve it graphically.
SOLUTION:
PART A: Find 10th term
a = 3, d = 4
$T_{10} = 3 + (10-1) \times 4$
= 3 + 36
= 39
PART B: Form equation
Roots: 2 and 5

```
Sum: 2 + 5 = 7
Product: 2 × 5 = 10
But we want constant = 39
So multiply: (x-2)(x-5) × 3.9
Or use: x² - 7x + 39 (adjusted)
Actually, standard form:
(x-2)(x-5) = 0
x² - 7x + 10 = 0
If constant should be 39:
Need different roots or approach
Let's use 39 as y-intercept:
y = x² - 7x + 39
PART C: Graph (would show parabola)
Topics Used:
✓ Arithmetic progression
✓ Quadratic equations
✓ Graphing
✓ Problem interpretation
```

C. Exam Technique and Tips

Diagram 144: Success Strategies

243244

```
BEFORE THE EXAM:
┌─────────────────────────────────┐
│ ✓ Review all formula sheets
│
│ ✓ Practice past questions
│
│ ✓ Identify weak areas
│
│ ✓ Study with examples
│
│ ✓ Get adequate sleep
│
│ ✓ Prepare materials
│
└─────────────────────────────────┘
DURING THE EXAM:
┌─────────────────────────────────┐
│ 1. READ questions carefully
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│ 2. PLAN time allocation
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│ 3. SHOW all working
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│ 4. CHECK answers
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│ 5. ANSWER all questions
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TIME MANAGEMENT:
50 marks exam in 90 minutes
= 1.8 minutes per mark
5-mark question → 9 minutes
10-mark question → 18 minutes
Leave time for checking!
COMMON MISTAKES TO AVOID:
✗ Not showing working
✗ Calculation errors
✗ Wrong formula
✗ Skipping steps
✗ Not verifying answers
✗ Poor presentation
✗ Leaving questions blank
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Comprehensive Practice Questions:

Section A: Standard Form and Approximation1. Express 0.00567 in standard form.
2. Round 3.14159 to 4 significant figures.
3. Calculate percentage error when π is approximated as 3.14.

Section B: Logarithms
4. Use logarithm tables to find log 345.
5. Evaluate using logarithms: 45.6 2.3 ÷ 8.4
6. Find the value of $(3.5)^4$ using logarithms.

Section C: Sequences
7. Find the 25th term of: 5, 9, 13, 17, ...
8. Calculate the sum of first 20 terms of: 3, 7, 11, 15, ...
9. The 5th term is 18 and 12th term is 46. Find first term and common difference.

Section D: Quadratic Equations
10. Solve by factorization: $x^2 - 7x + 12 = 0$
11. Solve by completing square: $x^2 + 6x - 7 = 0$
12. Use quadratic formula: $2x^2 - 5x - 3 = 0$
13. Form equation with roots 4 and -2.
14. If one root of $x^2 + kx - 6 = 0$ is 3, find k and other root.

Section E: Simultaneous Equations
15. Solve by elimination: $3x + 2y = 13$ $2x - y = 4$
16. Solve by substitution: $y = 2x - 1$ $3x + y = 14$
17. Solve: $y = x^2 - 2x$ $y = x + 4$

Section F: Graphical Solutions
18. Draw $y = 2x - 3$ for $-1 \leqslant x \leqslant 3$.
19. Draw $y = x^2 - 4x + 3$ for $0 \leqslant x \leqslant 4$. Find vertex.
24520. Solve graphically: $y = x + 1$ $y = 5 - x$

Mock Examination Questions:

SECTION A: Objective Questions (20 marks)

Diagram 145: Sample Objective Questions

1. Express 0.00456 in standard form
A. 4.56×10^{-3}
B. 4.56×10^{-2}
C. 45.6×10^{-4}
D. 0.456×10^{-2}
2. The 7th term of AP 3, 7, 11, ... is
A. 27
B. 28
C. 29
D. 31
3. The roots of $x^2 - 5x + 6 = 0$ are
A. 2 and 3
B. -2 and -3
C. 1 and 6
D. -1 and -6
4. $\log(M \times N)$ equals
A. $\log M \times \log N$
B. $\log M + \log N$
C. $\log M - \log N$
D. $\log M \div \log N$
5. If gradient of line is 3 and
y-intercept is -2, equation is
A. $y = 3x - 2$
B. $y = -2x + 3$
C. $y = 3x + 2$
D. $y = 2x - 3$
[Continue with 15 more questions...]

SECTION B: Theory Questions (80 marks)

246**Question 1 (10 marks):** (a) Solve by factorization: $x^2 - 8x + 15 = 0$ (b) Verify your answer

Question 2 (12 marks): The 3rd term of an AP is 14 and the 9th term is 38. (a) Find the first term and common difference (b) Find the 20th term (c) Calculate the sum of first 15 terms

Question 3 (15 marks): Solve the simultaneous equations: $3x + 2y = 16$ $5x - y = 9$ (a) By elimination method (b) By substitution method (c) Verify both give same answer

Question 4 (13 marks): Draw the graphs of: $y = x^2 - 2x - 3$ $y = x + 1$ for $-2 \leq x \leq 4$ Use your graph to solve $x^2 - 2x - 3 = x + 1$

Question 5 (10 marks): A shop sells pens at ₦50 each and books at ₦120 each. (a) Write equation for total cost of x pens and y books (b) If total cost is ₦680 and 4 pens were bought, find number of books (c) If 5 items were bought for ₦480, find how many of each

Question 6 (10 marks): Use logarithm tables to evaluate: $(34.5 \times 2.8) \div \sqrt{156}$

Question 7 (10 marks): The perimeter of rectangle is 34 cm. Its length is 3 cm more than width. (a) Form two equations (b) Find the dimensions (c) Calculate the area

Final Revision Checklist:

Diagram 146: Pre-Exam Checklist

TOPICS TO REVIEW:

□ Standard form conversions
□ Rounding to significant figures
□ Percentage error calculations
□ Logarithm laws
□ Using log tables
□ AP formulas (nth term and sum)
□ Factorizing quadratics
□ Completing the square
□ Quadratic formula
□ Sum and product of roots
□ Elimination method
□ Substitution method
□ Linear-quadratic systems
□ Plotting linear graphs
□ Drawing parabolas
□ Reading intersection points
□ Word problem strategies
□ Verification methods
247SKILLS TO MASTER:
□ Accurate calculations
□ Clear working
□ Proper labeling
□ Graph drawing
□ Time management
□ Question interpretation
□ Answer verification
MATERIALS NEEDED:
□ Mathematical set
□ Calculator
□ Graph paper
□ Ruler
□ Pencil and eraser
□ Exam admission

Class Activity:

Activity 1: Speed Revision Quiz

Teams compete answering quick-fire questions from all topics.

Activity 2: Error Correction

Find and correct errors in provided solutions.

Activity 3: Peer Teaching

Students teach each other difficult concepts.

Final Assignment:

Complete past examination paper under timed conditions. Mark and identify areas needing more work.