SS1 MATHEMATICS LESSON NOTES

FIRST TERM SCHEME OF WORK

1. Number bases

2. Modular Arithmetic

3. Indices

4. Standard form

5. Logarithm

6. Sets

7. Sets operations

9.Simple equations

WEEK 1: NUMBER BASE SYSTEMS

Concept: Understanding How Numbers Are Represented in Different Counting Systems

1. INTRODUCTION: BEYOND BASE 10

Welcome to SS1 Mathematics! Our first topic is the **Number Base System -** the fundamental framework that determines how we write, interpret, and calculate with numbers.

What is a Number Base?

A **number base** (or **radix**) is the number of unique digits, including zero, used to represent numbers in a positional numeral system.

Base	Name	Digits Used	Visual Representation
2	Binary	0, 1	● ○ (On/Off)
8	Octal	0 – 7	
10	Decimal	0 – 9	(Our fingers)

16	Hexadecimal	0 – 9, A – F	(Web colors)

The "Reset" Concept: The base number tells us when counting "resets" to the next place value. For example:

- **Base 10**: After 9 comes **10** (1 group of ten + 0 ones)
- **Base 2**: After 1 comes **10** (1 group of two + 0 ones)
- **Base 5**: After 4 comes **10** (1 group of five + 0 ones)

Real-World Connections

Diagram 1: Why Different Bases Exist

```
Human Counting (Base 10)    Computer Circuits (Base 2)


     10 Fingers            ● On/Off States
      |                          |
      ▼                          ▼
 0-9 Digits                 0-1 Bits
      |                          |
      ▼                          ▼
  Place Values             Binary System
```

Practical Applications:

- **Base 10 (Decimal)**: Everyday counting, money, measurements
- **Base 2 (Binary)**: Computer processors, digital storage
- **Base 16 (Hexadecimal)**: Web design (#FF0000 = red), memory addresses
- **Base 60**: Time (60 seconds = 1 minute), angles (60' = 1°)

Core Concept: Place Value Expansion

The General Formula: For any number $(b.e)$ in base x:

$$(\times^{2}) + (b \times^{1}) + (\times^{0}) + (\times^{-1}) + (e \times^{-2}) + (\times^{-3})$$

Visual Place Value Chart:

```
Base x Number:  (  2   4   3  .  1   2  )₅
                ┌───┬───┬───┬───┬───┬───┐
Place Values:  │5² │5¹ │5⁰ │    │5⁻¹│5⁻²│
               │25 │ 5 │ 1 │ . │0.2│0.04│
                └───┴───┴───┴───┴───┴───┘
```

2. CONTENT

A. CONVERSION FROM OTHER BASES TO BASE 10

Method: Polynomial Expansion

Step-by-Step Procedure:

1. Write the number and identify its base
2. Label each digit with its place value (powers of the base)
3. Multiply each digit by its place value
4. Sum all the products

Worked Example 1.1: Convert $(243)_5$ to Base 10

Visual Expansion:

```
  (2     4     3)₅
   ↓     ↓     ↓
 2×5² + 4×5¹ + 3×5⁰
   ↓     ↓     ↓
 2×25 + 4×5  + 3×1
   ↓     ↓     ↓
   50  + 20  + 3  = 73₁₀
```

Calculation Table:

Digit	Place Value	Power	Value	Cumulative
2	25	5^2	$2 \times 25 = 50$	50
4	5	5^1	$4 \times 5 = 20$	70
3	1	5^0	$3 \times 1 = 3$	**73**

✓ **Answer: $(243)_5 = 73_{10}$**

Worked Example 1.2: Convert $(11011)_2$ to Base 10

Binary Place Value Chart:

```
1     1     0     1     1₂
↓     ↓     ↓     ↓     ↓
2⁴    2³    2²    2¹    2⁰
16    8     4     2     1
↓     ↓     ↓     ↓     ↓
16 + 8   + 0   + 2   + 1 = 27₁₀
```

✅ **Answer: $(11011)_2 = 27_{10}$**

Worked Example 1.3: Convert $(12.34)_5$ to Base 10

Fractional Expansion:

```
Integer Part:        Fractional Part:
(1    2)₅            (3    4)₅
 ↓    ↓               ↓    ↓
1×5¹ + 2×5⁰         3×5⁻¹ + 4×5⁻²
 ↓    ↓               ↓    ↓
1×5 + 2×1           3×0.2 + 4×0.04
 ↓    ↓               ↓    ↓
 5   + 2 = 7         0.6 + 0.16 = 0.76
```

✅ **Answer: $(12.34)_5 = 7.76_{10}$**

B. CONVERSION FROM BASE 10 TO OTHER BASES

Method 1: Repeated Division (for Integer Part)

Algorithm:

1. Divide the number by the target base
2. Record the remainder
3. Use the quotient for the next division
4. Repeat until quotient becomes 0
5. Read remainders from bottom to top

Diagram: The Division Ladder

```
     Remainders
        ↑
       [1] ← Last division
      ÷ 2
       [1]
      ÷ 2
       [0]
59 → ÷ 2
       [1]
      ÷ 2
       [1]
      ÷ 2
       [1] ← First remainder
```

Worked Example 1.4: Convert 59_{10} to Base 2

Division Process:

Step	Division	Quotient	Remainder	Note
1	59 ÷ 2	29	**1**	(LSB) Least Significant Bit
2	29 ÷ 2	14	**1**	
3	14 ÷ 2	7	**0**	
4	7 ÷ 2	3	**1**	

Step	Division	Quotient	Remainder	Note
5	3 ÷ 2	1	**1**	
6	1 ÷ 2	0	**1**	(MSB) Most Significant Bit

Reading from bottom to top: 111011_2

✅ **Answer: $59_{10} = 111011_2$**

Verification: 1×32 + 1×16 + 1×8 + 0×4 + 1×2 + 1×1 = 59 ✓

Method 2: Repeated Multiplication (for Fractional Part)

Algorithm:

1. Multiply the fraction by the target base
2. Record the integer part of the result
3. Continue with the fractional part
4. Stop when fractional part becomes 0 or pattern repeats

Worked Example 1.5: Convert 0.375_{10} to Base 2

Multiplication Process:

Step	Operation	Integer	Fraction	Binary Digit
1	0.375 × 2 = 0.75	**0**	0.75	0
2	0.75 × 2 = 1.5	**1**	0.5	1
3	0.5 × 2 = 1.0	**1**	0.0	1

Reading from top to bottom: 0.011_2

✅ **Answer: $0.375_{10} = 0.011_2$**

Worked Example 1.6: Convert 26.8_{10} to Base 5

Integer Part (26_{10}):

```
26 ÷ 5 = 5 remainder 1
 5 ÷ 5 = 1 remainder 0
 1 ÷ 5 = 0 remainder 1
```

↑ Read bottom to top: **101_5**

Fractional Part (0.8_{10}):

```
0.8 × 5 = 4.0 → integer = 4, fraction = 0
```

→ **0.4_5**

✅ **Answer: $26.8_{10} = 101.4_5$**

C. ARITHMETIC OPERATIONS IN DIFFERENT BASES

Golden Rule: All operations follow the same logic as base 10, but the "carry" and "borrow" values equal the base.

Addition in Different Bases

Key Concept: When sum ≥ base, carry over to next column

Worked Example 1.7: Add $243_5 + 134_5$

Step-by-Step Solution:

```
  2 4 3₅
+ 1 3 4₅
────────
```

Column-by-column calculation:

Column	Calculation	Base 5 Conversion	Write	Carry
Units	3 + 4 = 7	7 ÷ 5 = 1 remainder 2	2	1
Fives	4 + 3 + 1 = 8	8 ÷ 5 = 1 remainder 3	3	1
Twenty-fives	2 + 1 + 1 = 4	4 < 5	4	0

✅ **Answer: 432_5**

Verification: $243_5 = 73_{10}$, $134_5 = 44_{10}$, $73 + 44 = 117_{10} = 432_5$ ✓

Subtraction in Different Bases

Key Concept: Borrowing gives you the BASE value, not 10

Worked Example 1.8: Subtract $412_8 - 153_8$

Detailed Borrowing Process:

```
  4 1 2₈      Step 1: 2 < 3, borrow from middle column
- 1 5 3₈      Middle column: 1 becomes 0, units get 8 (base value)
────────      Now: 2 + 8 = 10, 10 - 3 = 7

  4 0 12₈     Step 2: 0 < 5, borrow from left column
- 1 5 3₈      Left column: 4 becomes 3, middle gets 8
────────      Now: 0 + 8 = 8, 8 - 5 = 3

 3 8 12₈      Step 3: 3 - 1 = 2
-1 5 3₈
────────
  2 3 7₈
```

✅ **Answer: 237_8**

Multiplication in Different Bases

Key Concept: Use multiplication tables in the given base

Worked Example 1.9: Multiply $110_2 \times 11_2$

Long Multiplication:

```
    1 1 0₂
  ×   1 1₂
  ────────
    1 1 0   ← 110 × 1
+ 1 1 0     ← 110 × 10 (shifted left)
─────────
  1 0 0 1 0₂
```

Step Verification:

- In base 2: $1 \times 110 = 110$
- In base 2: $10 \times 110 = 1100$ (equivalent to shifting left)
- Addition: $110 + 1100 = 10010_2$

✅ **Answer: 10010_2**

Decimal Check: $110_2 = 6_{10}$, $11_2 = 3_{10}$, $6 \times 3 = 18_{10} = 10010_2$ ✓

D. APPLICATIONS IN COMPUTING AND DIGITAL SYSTEMS

Diagram: How Computers Use Different Bases

```
Raw Data:         01011010 10101101 11110000
Grouped (Hex):      5   A    A   D    F   0
Hexadecimal:        5A AD F0
Memory Address:     0x5AADF0
```

Practical Applications Table:

Base	Primary Use	Real-World Example
Binary (2)	CPU operations, file storage	Computer processors, RAM
Octal (8)	Unix permissions, legacy systems	Linux: chmod 755
Hexadecimal (16)	Memory addresses, color codes	CSS: #FF5733 = RGB(255,87,51)

Color Code Example:

```
# FF 57 33₁₆
 |  |  |
 |  |  └─ Red component: 33₁₆ = 51₁₀
 |  └─ Green component: 57₁₆ = 87₁₀
 └─ Blue component: FF₁₆ = 255₁₀
```

3. REVISION QUESTIONS (WEEK 1)

1. Why do humans naturally use base 10, while computers use base 2?
2. Explain why the number $(396)_7$ is invalid.
3. Describe the difference between the repeated division and repeated multiplication methods.
4. How does borrowing work in base 8 compared to base 10?
5. Give three real-world applications of hexadecimal numbers.

4. ASSESSMENT (WEEK 1)

EVALUATION (IN-CLASS)

1. Convert $(324)_5$ to base 10.
2. Convert $(127)_{10}$ to base 8.
3. Convert $(0.625)_{10}$ to base 2.
4. Calculate $(132)_4 + (211)_4$.

ASSIGNMENT (TAKE-HOME)

1. Convert $(2A9)_{16}$ to base 10. (Hint: A = 10, B = 11, C = 12, D = 13, E = 14, F = 15)
2. Convert $(158)_{10}$ to base 5.
3. Convert $(44.75)_{10}$ to base 2.

Quick Reference: Base Conversion Cheat Sheet

To Decimal: Expand using place values **From Decimal (Integer):** Repeated division, read remainders upward **From Decimal (Fraction):** Repeated multiplication, read integers downward **Between Binary/Hex:** Group binary digits in 4s ($2^3 = 8$, $2^4 = 16$)

Common Bases in Computing:

- 1 byte = 8 bits = 2 hexadecimal digits
- 1 kibibyte = 1024 bytes = 2^{10} bytes
- RGB colors: 00 to FF for each component (0-255 in decimal)

Answers to Assessment

In-Class:

1. 89
2. 45
3. 177

Take-Home:

1. 681
2. 1113
3. 101100.11
4. 214

WEEK 2: MODULAR ARITHMETIC

Concept: The Mathematics of Cycles and Remainders

1. INTRODUCTION: THE CLOCK AS A MATHEMATICAL MODEL

Imagine a clock. If it's 9 o'clock now, and you add 5 hours, it becomes 2 o'clock, not 14 o'clock. This "wrapping around" behavior is the essence of **Modular Arithmetic**.

Core Idea: In Modular Arithmetic, we are only interested in the **remainder** after division by a fixed number called the **modulus**. The clock's modulus is 12.

Let's formalize our clock example:

- Normal Arithmetic: $9 + 5 = 14$
- Modular Arithmetic (mod 12): $14 \div 12 = 1$ remainder **2**.

So, we write: $\mathbf{9 + 5 \equiv 2 \pmod{12}}$ This is read as: **"9 plus 5 is congruent to 2, modulo 12."**

⏰ Visual Foundation: The Clock & Number Line Analogy

Diagram 1: The Classic Clock

```
         12
     11       1
    10          2
     9          3
      8       4
         7
```

Each hour we move, we cycle around the clock face. After 12, we return to 1.

Diagram 2: The Circular Number Line (Mod 5)

```
        0
    4       1
     3     2
```

This is a more general "clock" for a modulus of 5. The numbers 0, 1, 2, 3, 4 are equally spaced. If you start at 2 and add 4, you go (2→3→4→0→1), landing on **1**. So, 2 + 4 ≡ 1 (mod 5).

Diagram 3: Linear Number Line Wrapping into a Circle

```
... -5, 0, 5, 10 ...  all map to → 0 (mod 5)
... -4, 1, 6, 11 ...  all map to → 1 (mod 5)
... -3, 2, 7, 12 ...  all map to → 2 (mod 5)
... -2, 3, 8, 13 ...  all map to → 3 (mod 5)
... -1, 4, 9, 14 ...  all map to → 4 (mod 5)
```

Note: Every number on the number line "wraps around" to one of the five residues {0, 1, 2, 3, 4} in a modulo 5 system. These groups are called **congruence classes**.

2. CONTENT

A. The Formal Concept of Congruence

Two numbers, **a** and **b**, are said to be **congruent modulo m** if their difference (a - b) is exactly divisible by the modulus **m**.

Mathematical Definition: a ≡ b (mod m) if and only if **m | (a - b)** (meaning "m divides (a - b)").

Example: Is 17 ≡ 5 (mod 6)? Check: 17 - 5 = 12. Is 12 divisible by 6? Yes (12 ÷ 6 = 2). ✅ Therefore, 17 ≡ 5 (mod 6).

B. Finding the Remainder (The Value Modulo m)

To find **a (mod m)**:

1. Divide **a** by **m**.
2. The **remainder** (which must be between 0 and m-1) is your result.

Worked Illustrations

Example	Operation & Calculation	Explanation & Conclusion
25 (mod 4)	25 ÷ 4 = 6 with a remainder of **1**	✅ 25 = 6×4 + **1**, so **25 ≡ 1 (mod 4)**
10 (mod 7)	10 ÷ 7 = 1 with a remainder of **3**	✅ 10 = 1×7 + **3**, so **10 ≡ 3 (mod 7)**

C. Handling Negative Numbers

For negative numbers, we want a **positive remainder**. We achieve this by repeatedly **adding the modulus** until we get a non-negative number less than m.

Example: Find -14 (mod 3) Think: "What number between 0 and 2 is equivalent to -14?" -14 + 3 = -11 -11 + 3 = -8 -8 + 3 = -5 -5 + 3 = **-2 -2 + 3 = 1** ✅ **Answer: -14 ≡ 1 (mod 3)**

Quick Check: -14 ÷ 3 = -5 remainder 1.

3. OPERATIONS IN MODULAR ARITHMETIC

The power of modular arithmetic is that we can perform operations (+, -, ×) on the "smaller" remainders instead of the large original numbers.

Key Principle: You can reduce numbers using the modulus *before* or *after* performing operations. The result will be congruent.

Operation	General Rule	Efficient Method
Addition	(a + b) mod m	Find (a mod m) and (b mod m), add them, then take mod m.
Subtraction	(a - b) mod m	Find (a mod m) and (b mod m), subtract, then take mod m. (If result is negative, add m).
Multiplication	(a × b) mod m	Find (a mod m) and (b mod m), multiply them, then take mod m.

Note: This works because of the distributive properties of modular arithmetic. For example, if $a = mq_1 + r_1$ and $b = mq_2 + r_2$, then $a + b = m(q_1+q_2) + (r_1+r_2)$. The term with 'm' disappears when we take mod m, leaving only (r_1+r_2) mod m.

4. WORKED EXAMPLES (WITH DEEPER EXPLANATION)

Example 2.1: Evaluate 47 (mod 6)

- **Solution:** 47 ÷ 6 = 7 remainder **5**.
- ✅ **Answer: 47 ≡ 5 (mod 6)**. This means on a "clock" of size 6, 47 o'clock is the same as 5 o'clock.

Example 2.2: Evaluate (38 + 29) (mod 3)

- **Method 1 — Add First:** 38 + 29 = 67. 67 ÷ 3 = 22 remainder **1**.
- **Method 2 — Mod First (More Efficient):** 38 (mod 3) = 2 (since 38 ÷ 3 = 12 r2) 29 (mod 3) = 2 (since 29 ÷ 3 = 9 r2) (2 + 2) = 4. 4 (mod 3) = **1**.
- ✅ **Answer: 1 (mod 3)**. Both methods agree!

Example 2.3: Compute (15 × 22) (mod 7)

- **Method 1 — Multiply First:** 15 × 22 = 330. 330 ÷ 7 = 47 remainder **1**.
- **Method 2 — Mod First (Much Easier!):** 15 (mod 7) = **1**. 22 (mod 7) = **1**. (1 × 1) = 1. 1 (mod 7) = **1**.
- ✅ **Answer: 1 (mod 7)**. This shows the power of simplification.

Example 2.4: Evaluate (25 - 12) (mod 8)

- **Solution using mod first:** 25 (mod 8) = **1**. 12 (mod 8) = **4**. (1 - 4) = -3. Since the result is negative, we add the modulus: -3 + 8 = **5**.
- ✅ **Answer: 5 (mod 8)**.

Example 2.5: Evaluate (83 × 56 + 12) (mod 9)

- **Solution using mod first:** 83 (mod 9): 83 ÷ 9 = 9 r2 → **2**. 56 (mod 9): 56 ÷ 9 = 6 r2 → **2**. 12 (mod 9): 12 ÷ 9 = 1 r3 → **3**. Now compute: (2 × 2 + 3) = (4 + 3) = 7. 7 (mod 9) = **7**.
- ✅ **Answer: 7 (mod 9)**.

5. APPLICATIONS TO DAILY LIFE & COMPUTER SCIENCE

A. Days of the Week — A Mod 7 System

We assign: Sun=0, Mon=1, Tue=2, Wed=3, Thu=4, Fri=5, Sat=6. **Q:** If today is Wednesday (3), what day is 90 days from now?

- (3 + 90) mod 7 = 93 mod 7.
- 93 ÷ 7 = 13 remainder **2**.
- ✅ **Answer: 2 → Tuesday.**

B. Clock Time — Mod 12 or Mod 24

Q: If it's 7:00 PM (19:00) now, what time will it be in 40 hours?

- (19 + 40) mod 24 = 59 mod 24.
- 59 ÷ 24 = 2 remainder **11**.
- ✅ **Answer: 11:00 AM.**

C. Cryptography & Computer Science

- **ISBN Check Digits:** The last digit of a book's ISBN code is calculated using mod 11 to detect errors in typing or scanning.
- **Cyclic Redundancy Checks (CRCs):** Used in networks and data storage (like on a CD) to detect accidental changes to raw data. It relies heavily on polynomial division modulo 2.
- **Ciphers:** Simple ciphers like the Caesar cipher are just modular arithmetic on the alphabet (mod 26).

6. REVISION QUESTIONS (WEEK 2)

1. What is another name for modular arithmetic? *(Hint: Think about what we are left with.)*
2. Explain in your own words what **a ≡ b (mod m)** means.
3. Describe the process for finding the positive value of a negative number like -17 (mod 5).

7. ASSESSMENT (WEEK 2)

EVALUATION (IN-CLASS)

1. Evaluate 100 (mod 9).
2. Evaluate -20 (mod 6).
3. Calculate (45 + 52) (mod 5).

ASSIGNMENT (TAKE-HOME)

1. Find the simplest positive value of **x**: a. x ≡ 55 (mod 8) b. x ≡ -30 (mod 7)
2. Calculate (48 - 19) (mod 3).
3. Evaluate (12 × 11 × 10) (mod 6).

Answers to Assessment

In-Class:

1. 1
2. 4
3. 2

Take-Home:

1. a) 7, b) 5
2. 2
3. 0

WEEK 3: INDICES (POWERS AND EXPONENTS)

Concept: The Mathematics of Repeated Multiplication and Exponential Growth

1. INTRODUCTION: THE POWER OF REPETITION

Indices (also called exponents or powers) are mathematical shorthand for repeated multiplication. They help us express very large and very small numbers efficiently.

Basic Terminology:

```
2 × 2 × 2 × 2 × 2 = 2⁵
    |        |      |
    |        |      └─ Exponent (Power)
    |        └──────── Base
    └───────────────── Repeated Multiplication
```

Read as: "Two raised to the power of five" or "Two to the fifth power"

Real-World Exponential Growth

Diagram 1: Bacterial Growth (Doubling Every Hour)

```
Hour 0: ●                    = 1 = 2⁰
Hour 1: ●●                   = 2 = 2¹
Hour 2: ●●●●                 = 4 = 2²
Hour 3: ●●●●●●●●             = 8 = 2³
Hour 4: ●●●●●●●●●●●●●●●●     = 16 = 2⁴
```

Diagram 2: Compound Interest Growth

```
Year 0: ₦1,000 = 1000 × 2⁰
Year 1: ₦2,000 = 1000 × 2¹
Year 2: ₦4,000 = 1000 × 2²
```

```
Year 3: ₦8,000 = 1000 × 2³
      Exponential growth curve
```

2. CONTENT

A. STANDARD FORM (SCIENTIFIC NOTATION)

Standard form helps us write extremely large or extremely small numbers efficiently.

Format: $\times 10$ where $1 \leq \; < 10$ and is an integer

Visual Guide: Decimal Point Movement

Large Numbers → Positive Exponents:

```
4,750,000 = 4.75 × 10⁶
   |         |      |
   |         |      └─ Decimal moved 6 places LEFT
   |         └──────── Number between 1 and 10
   └────────────────── Original number
```

Small Numbers → Negative Exponents:

```
0.00032 = 3.2 × 10⁻⁴
  |        |      |
  |        |      └─ Decimal moved 4 places RIGHT
  |        └──────── Number between 1 and 10
  └───────────────── Original number
```

Memory Aid:

```
← LEFT = POSITIVE exponent    RIGHT → = NEGATIVE exponent
Large numbers (4,750,000)    Small numbers (0.00032)
```

B. THE SEVEN LAWS OF INDICES

Visual Summary: The Index Law Family

```
Addition Family (Multiplication)    Subtraction Family (Division)
      aᵐ × aⁿ = aᵐ⁺ⁿ                          aᵐ ÷ aⁿ = aᵐ⁻ⁿ
      ↓   ↓   ↓                                ↓   ↓   ↓
    Same base → ADD exponents         Same base → SUBTRACT exponents


Multiplication Family (Power of Power)
          (aᵐ)ⁿ = aᵐⁿ
          ↓     ↓
      Multiply the exponents


Special Cases Family:
a⁰ = 1 (Zero Power)           a⁻ⁿ = 1/aⁿ (Negative Power)
a¹/ⁿ = ⁿ√a (Root Power)       aᵐ/ⁿ = (ⁿ√a)ᵐ (Combined Power)
```

Detailed Law Breakdown:

Law	Rule	Example	Visual Explanation
1. Multiplication	${}^{m}\times{}^{n} = {}^{m+n}$	${}^{5}\times{}^{3} = {}^{8}$	$\cdots\cdot \times \cdot\cdot = {}^{8}$
2. Division	${}^{m}\div{}^{n} = {}^{m-n}$	${}^{9}\div{}^{2} = {}^{7}$	Cancel 2 y's: $\cdots\cdots = {}^{7}$
3. Power of Power	$({}^{m})^{n} = {}^{mn}$	$({}^{3})^{4} = {}^{12}$	$(\cdot\cdot)\times(\cdot\cdot)\times(\cdot\cdot)\times(\cdot\cdot) = {}^{12}$
4. Zero Power	${}^{0} = 1$	$7^0 = 1$	Pattern: $7^3 = 343, 7^2 = 49, 7^1 = 7, 7^0 = 1$
5. Negative Power	${}^{-n} = \frac{1}{{}^{n}}$	$3^{-2} = \frac{1}{9}$	Opposite of positive power

Law	Rule	Example	Visual Explanation
6. Fractional Power	1/ = √	$64^{1/3} = 4$	"What number cubed equals 64?"
7. Combined Fractional	/ = ( )√	$27^{2/3} = 9$	Cube root of 27 = 3, then square = 9

Conceptual Understanding:

Why does $a^0 = 1$?

```
Pattern Observation:
3⁴ = 81
3³ = 27  (81 ÷ 3)
3² = 9   (27 ÷ 3)
3¹ = 3   (9 ÷ 3)
3⁰ = 1   (3 ÷ 3)  ← Continuing the pattern!
```

Negative Exponents as Reciprocals:

```
2³ = 8          2⁻³ = 1/8
|               |
└→ Positive     └→ Reciprocal
   exponent        of positive
```

C. SOLVING EXPONENTIAL EQUATIONS

Golden Rule: If = and ≠ 0,1, then =

Strategy 1: Make Bases the Same

```
Example: 8 = 2³, 27 = 3³, 16 = 4² = 2⁴
Common conversions:
4 = 2²      9 = 3²      25 = 5²
8 = 2³      27 = 3³     125 = 5³
16 = 2⁴     81 = 3⁴     625 = 5⁴
```

Strategy 2: Substitution for Quadratic Forms When you have $^2 + b\cdot + \ = 0$, let $\ =$

---

3. DETAILED WORKED EXAMPLES

Example 3.1: Simplify $\left(\frac{81}{16}\right)^{-3/4}$

Step-by-Step Solution:

```
Step 1: Handle negative exponent (flip fraction)
(81/16)⁻³/⁴ = (16/81)³/⁴

Step 2: Apply fractional exponent (root then power)
= (⁴√(16/81))³

Step 3: Calculate fourth roots
⁴√16 = 2, ⁴√81 = 3
= (2/3)³

Step 4: Cube the fraction
= 8/27
```

Visual Process:

```
(81/16)⁻³/⁴ → [FLIP] → (16/81)³/⁴ → [ROOT] → (2/3)³ → [POWER] → 8/27
    |            |            |            |         |         |
    |            |            |            |         |         └─ Final Answer
    |            |            |            |         └─ Apply exponent 3
    |            |            |            └─ Take 4th root of numerator &
denominator
    |            |            └─ New base after flipping
    |            └─ Negative exponent means reciprocal
    └─ Original expression
```

✅ **Answer: 8/27**

Example 3.2: Simplify $\frac{(^{2-1})^{3}}{45}$

Step-by-Step Solution:

Step 1: Apply power to numerator

$(x^2y^{-1})^3 = x^6y^{-3}$

Step 2: Write the full expression

$= x^6y^{-3} / (x^4y^5)$

Step 3: Apply division law (subtract exponents)

$x^{6-4}\ y^{-3-5} = x^2\ y^{-8}$

Step 4: Handle negative exponent

$= x^2 / y^8$

Exponent Arithmetic:

Numerator exponents: x^6y^{-3}

Denominator exponents: x^4y^5

Result: $x^{6-4}\ y^{-3-5} = x^2\ y^{-8}$

✅ **Answer: x^2/y^8**

Example 3.3: Solve $5 = 125$

Solution:

$5^x = 125$

$5^x = 5^3$ ← Express 125 as power of 5

$x = 3$ ← Equate exponents (same base)

✅ **Answer: $x = 3$**

Example 3.4: Solve $9^{-1} = 27^{+2}$

Step-by-Step Solution:

Step 1: Express both sides with base 3

$9^{x-1} = (3^2)^{x-1} = 3^{2(x-1)}$

$27^{x+2} = (3^3)^{x+2} = 3^{3(x+2)}$

Step 2: Equate the expressions

$3^{2(x-1)} = 3^{3(x+2)}$

Step 3: Equate exponents (same base)

$2(x - 1) = 3(x + 2)$

Step 4: Solve the linear equation

$2x - 2 = 3x + 6$

$2x - 3x = 6 + 2$

$-x = 8$

$x = -8$

Base Conversion Visualization:

$9^{x-1} = 27^{x+2}$

$\downarrow \qquad \downarrow$

$(3^2)^{x-1} = (3^3)^{x+2}$

$\downarrow \qquad \downarrow$

$3^{2x-2} = 3^{3x+6}$

✅ **Answer: x = -8**

Example 3.5: Solve $4 - 3(2) - 4 = 0$

Step-by-Step Solution:

Step 1: Recognize that $4^x = (2^x)^2$

Let $y = 2^x$, then $4^x = y^2$

Step 2: Substitute

$y^2 - 3y - 4 = 0$

Step 3: Factor the quadratic

$(y - 4)(y + 1) = 0$

Step 4: Solve for y

$y = 4$ or $y = -1$

Step 5: Back-substitute $y = 2^x$

Case 1: $2^x = 4 \rightarrow 2^x = 2^2 \rightarrow x = 2$

Case 2: $2^x = -1 \rightarrow$ No solution ($2^x > 0$ for all real x)

Substitution Method Visualization:

```
Original: 4ˣ - 3(2ˣ) - 4 = 0
          │        │
          │        └─ This is y
          └─ This is y²

Becomes: y² - 3y - 4 = 0  ← Much easier to solve!
```

✅ **Answer: $x = 2$**

4. REVISION QUESTIONS (WEEK 3)

1. Explain the conceptual difference between $^{-}$ and $^{1/}$ using examples.
2. Convert 0.000507 to standard form and explain each step.
3. Using the pattern method, show why any non-zero number raised to power 0 equals 1.

5. ASSESSMENT (WEEK 3)

EVALUATION (IN-CLASS)

1. Write 5.67×10^{-4} as an ordinary number.
2. Simplify $\frac{{}^{3}b^{2}}{{}^{5}b^{\ -1}}$
3. Evaluate $\left(\frac{64}{9}\right)^{-1/2}$

ASSIGNMENT (TAKE-HOME)

1. Simplify $\sqrt{{}^{34}} \times ({}^{23})^{-1/2}$
2. Evaluate $\frac{5^{\times 25^{-1}}}{125^{+1}}$ as a power of 5
3. Solve for k: $16^{2-3} = 32^{1-}$

6. BONUS: POWERS OF 2 VISUALIZATION

Table: The Exponential Growth of Powers of 2

Power	Expression	Value	Real-World Equivalent
2^0	1	1	Single bacterium
2^1	2	2	Two coins

Power	Expression	Value	Real-World Equivalent
2^2	2 × 2	4	Four cardinal directions
2^3	2 × 2 × 2	8	Bits in a byte (almost)
2^4	2 × 2 × 2 × 2	16	Squares on chessboard (one side)
2^5	...	32	Adult human teeth
2^6	...	64	Squares on chessboard (total)
2^7	...	128	ASCII character set size

2^8	...	256	One byte (8 bits)
2^{10}	...	1,024	One kilobyte (in computing)

Negative Powers (The Shrinking Pattern):

Power	Expression	Value	Explanation
2^{-1}	$1 \div 2$	0.5	Half
2^{-2}	$1 \div 4$	0.25	Quarter
2^{-3}	$1 \div 8$	0.125	Eighth

Memory Aid: The Exponent Rules Song

```
When multiplying, add the powers high,
When dividing, subtract them by and by,
A power of a power, multiply you'll see,
Anything to zero power is one, you'll agree!
Negative power means flip it right,
Fractional power is a root, that's the light!
```

Answers to Assessment

In-Class:

1. 0.000567

2. $\frac{b^3}{2}$

$\frac{3}{8}$

Take-Home:

1. $1/25/2$
2. 5^{--4}
3. $= \frac{11}{7}$

WEEK 4: LOGARITHMS

Concept: The Inverse Operation of Exponentiation - Finding the Exponent

1. NTRODUCTION: THE MISSING PIECE IN EXPONENTIAL EQUATIONS

Last week, we studied indices (powers), where we raised a base to an exponent to get a number:

```
10³ = 1000
  |   |    |
  |   |    └─ Result
  |   └────── Exponent
  └────────── Base
```

This week, we study the **inverse operation** — logarithms. A logarithm answers the question:

"What power must I raise the base to, in order to get a given number?"

The Inverse Relationship: Logarithms vs. Exponents

Diagram 1: Inverse Operations Comparison

```
Addition ↔ Subtraction          Multiplication ↔ Division
    5 + 3 = 8                       5 × 3 = 15
    8 - 3 = 5                       15 ÷ 3 = 5


 Exponentiation ↔ Logarithm
    10³ = 1000                      log₁₀ 1000 = 3
    |   |    |                      |     |     |
    |   |    └─ Result              |     |     └─ Exponent
    |   └────── Exponent            |     └────── Result
    └────────── Base                └──────────── Base
```

Real-World Analogy: The Detective Story

```
CRIME SCENE: 10³ = 1000
We know: Base = 10, Result = 1000
MYSTERY: What was the exponent?

LOGARITHM DETECTIVE: "The exponent was 3!"
log₁₀ 1000 = 3
```

2. CONTENT

A. FORMAL DEFINITION OF LOGARITHMS

Mathematical Definition: If = , then = log

Where:

- → **base** (must be positive and ≠ 1)
- → **argument** (the result in exponential form)
- → **logarithm** (the exponent we're looking for)

Memory Aid: "The logarithm is the power!"

Visual: Converting Between Forms

```
Exponential Form → Logarithmic Form
aˣ = y    ⟺   logₐ y = x
 | | |          |  |  |
 | | └ result |  |  └ exponent
 | └── exponent |  └── result
 └──── base     └──── base
```

Examples 4.1: Converting Between Forms

Exponential Form	Logarithmic Form	Verbal Explanation
$10^2 = 100$	$\log_{10}100 = 2$	"10 to what power gives 100? The answer is 2"
$3^4 = 81$	$\log_3 81 = 4$	"3 to what power gives 81? The answer is 4"
$5^{-1} = \frac{1}{5}$	$\log_5\left(\frac{1}{5}\right) = -1$	"5 to what power gives 1/5? The answer is -1"
$16^{1/2} = 4$	$\log_{16} 4 = \frac{1}{2}$	"16 to what power gives 4? The answer is 1/2"

Fundamental Logarithm Identities

Identity 1: log = 1

Why? Because $a^1 = a$

Examples:

$\log_{10} 10 = 1$ (since $10^1 = 10$)

$\log_5 5 = 1$ (since $5^1 = 5$)

$\log_x x = 1$ (for any valid base x)

Identity 2: log1 = 0

Why? Because $a^0 = 1$

Examples:

$\log_{10} 1 = 0$ (since $10^0 = 1$)

$\log_7 1 = 0$ (since $7^0 = 1$)

$\log_x 1 = 0$ (for any valid base x)

Memory Aid: "Base to the power of the log equals the argument"

B. CHARACTERISTIC AND MANTISSA

Diagram 2: Anatomy of a Logarithm

```
log₁₀ 5000 = 3.6990
            |  ||||
            |  |||└─ Last decimal digit
            |  ||└── Thousandths
            |  |└─── Hundredths
            |  └──── Tenths
            └─────── Characteristic (integer part)
                     Mantissa (decimal part)
```

Understanding Characteristic from Standard Form:

Number	Standard Form	Characteristic	Reasoning
5000	5.0×10^3	3	Power of 10 is +3
0.00345	3.45×10^{-3}	-3	Power of 10 is -3

Bar Notation for Negative Characteristics:

Instead of writing -3.5378, we write: 3.5378 Read as: "Bar three point five three seven eight"

Examples:

-2.8549 = 2.8549

-1.2345 = 1.2345

-0.9876 = 0.9876

C. GRAPHS: EXPONENTIAL vs. LOGARITHMIC FUNCTIONS

Graph 1: Exponential Function $y = 10^x$

```
    y
    |
100 |               •
    |             /
    |           /
 10 |--------------•
    |       /
    |     /
   1 •---/
    | /
    |/
 ───┼───────┼──────── x
   -1      0       1
```

Key Features of $y = 10^x$:

- Passes through (0, 1) because $10^0 = 1$
- Never touches the x-axis (horizontal asymptote at y = 0)
- Rapid growth for positive x
- Slow decay for negative x

Graph 2: Logarithmic Function $y = \log_{10} x$

```
    y
    |
   1|       /•
    |     /
    |   /
   0 •─/───────
    |/   1   10
  -1|
    |
```

Key Features of $y = \log_{10} x$:

- Passes through (1, 0) because $\log_{10} 1 = 0$
- Never touches the y-axis (vertical asymptote at $x = 0$)
- Slow growth for $x > 1$
- Undefined for $x \leq 0$

Graph 3: Inverse Relationship

```
    y
    |
    |    y=10^x
    |      •/
    |     / •
    |    /   y=x
    |   / •
    |  / /
    | •/    y=log_10x
    |/
 ───┼───────┼───────  x
    |
```

The graphs are **mirror images** across the line $y = x$!

D. USING LOGARITHM AND ANTILOGARITHM TABLES

Method 1: Finding Logarithms

Step-by-Step Procedure:

1. Convert number to standard form: $= \times 10$
2. Characteristic =
3. Use log table with the digits of A to find mantissa
4. Combine: $\log =$ .mantissa

Worked Example 4.2: Find $\log_{10} 43.82$

Step 1: Standard Form

```
43.82 = 4.382 × 10¹ → Characteristic = 1
```

Step 2: Use Log Table

```
Look up 4.382 in log table:
- Find row 43, column 8 → 0.6415
- Find difference for 2 (last digit) → 0.0002
- Mantissa = 0.6415 + 0.0002 = 0.6417
```

Step 3: Combine

```
log₁₀ 43.82 = 1.6417
```

Visual Table Lookup:

```
Log Table Extract:
    0   1   2   ...   8   9
43  .6335 .6345 .6355 ... .6415 .6425
    ↑           ↑
    Row 43      Column 8 → 0.6415
                Difference for 2 → +0.0002
                Total mantissa = 0.6417
```

Worked Example 4.3: Find $\log_{10} 0.0716$

Step 1: Standard Form

```
0.0716 = 7.16 × 10⁻² → Characteristic = -2
```

Step 2: Use Log Table

Look up 7.16 in log table:

- Row 71, column 6 → 0.8549
- Mantissa = 0.8549

Step 3: Combine with Bar Notation

$\log_{10} 0.0716 = 2.8549$

Method 2: Finding Antilogarithms

Step-by-Step Procedure:

1. Separate characteristic and mantissa
2. Use antilog table with mantissa to find the digits
3. Use characteristic to place decimal point

Worked Example 4.4: Find antilog(3.9042)

Step 1: Separate

Characteristic = 3, Mantissa = 0.9042

Step 2: Use Antilog Table

Look up 0.9042 in antilog table:

- Row 90, column 4 → 8017
- Difference for 2 → +4 → 8021
- Digits = 8021

Step 3: Place Decimal Point

Characteristic = 3 → Decimal after 3 digits from left

8021 → 802.1? Wait, characteristic 3 means: $A \times 10^3$

So: $8.021 \times 10^3 = 8021$

✅ **Answer: 8021**

Worked Example 4.5: Find antilog($\bar{1}.6609$)

Step 1: Separate

Characteristic = -1, Mantissa = 0.6609

Step 2: Use Antilog Table

Look up 0.6609:

- Row 66, column 0 → 4571
- Difference for 9 → +9 → 4580
- Digits = 4580 → 4.580

Step 3: Place Decimal Point

Characteristic = -1 → $A \times 10^{-1} = 4.580 \times 10^{-1} = 0.4580$

✅ **Answer: 0.4580**

Memory Aid: Table Usage Flowchart

```
Finding LOG:                    Finding ANTILOG:
Number → Standard Form          Logarithm → Separate
       ↓                               ↓
Characteristic (from power)     Use mantissa in antilog table
       ↓                               ↓
Use A in log table → Mantissa   Get digits
       ↓                               ↓
Combine → Final answer          Use characteristic → Place decimal
```

3. REVISION QUESTIONS (WEEK 4)

1. Explain why logarithms and exponents are considered inverse operations, using a real-world analogy.
2. If $\log_b =$, write this in exponential form and explain what each variable represents.
3. Why is $\log_5 5 = 1$ and $\log_5 1 = 0$? Use the definition of logarithms to explain.

4. ASSESSMENT (WEEK 4)

EVALUATION (IN-CLASS)

1. Write $2^5 = 32$ in logarithmic form.
2. Write $\log_3 \left(\frac{1}{9}\right) = -2$ in exponential form.

3. Solve for x: log49 = 2

ASSIGNMENT (TAKE-HOME)

1. Solve for x: $\log_4 = 3$
2. Solve for x: $\log_3 27 =$
3. Using log tables, find:
    - $\log_{10} 602.4$
    - $\log_{10} 0.000199$

5. REAL-WORLD APPLICATIONS OF LOGARITHMS

Application 1: Richter Scale (Earthquake Magnitude)

```
Richter Scale is logarithmic: Each whole number increase = 10× energy
Magnitude 3 → 10³ joules
Magnitude 4 → 10⁴ joules (10 times more energy!)
Magnitude 5 → 10⁵ joules (100 times more than magnitude 3!)
```

Application 2: pH Scale (Acidity)

```
pH = -log₁₀[H⁺] where [H⁺] is hydrogen ion concentration
pH 3 → [H⁺] = 10⁻³ = 0.001
pH 4 → [H⁺] = 10⁻⁴ = 0.0001 (10 times less acidic!)
```

Application 3: Sound Intensity (Decibels)

```
dB = 10·log₁₀(I/I₀) where I is sound intensity
Whisper: 30 dB → 10³ times reference intensity
Normal talk: 60 dB → 10⁶ times reference intensity
```

Memory Poem:

```
Logarithms find the power, that's their special dower,
Inverse of exponents, their incredible power!
Characteristic tells us where the decimal points tower,
Mantissa gives the digits, making numbers not sour!
```

Answers to Assessment

In-Class:

1. $\log_2 32 = 5$
2. $3^{-2} = \frac{1}{9}$
3. $= 7$ (since $7^2 = 49$)

Take-Home:

1. $= 64$ (since $4^3 = 64$)
2. $= 3$ (since $3^3 = 27$)
3. a) ~2.7799, b) 4.2989

WEEK 6: EFFECTIVE USE OF LOGARITHM TABLES

Concept: Using Logarithms as a Human Calculator for Complex Operations

1. INTRODUCTION: THE PRE-CALCULATOR SUPERPOWER

Before digital calculators, scientists, engineers, and mathematicians used logarithm tables as their "human computers" to solve complex calculations efficiently.

Why Logarithms Transform Mathematics

Diagram 1: The Logarithmic Transformation Machine

```
Complex Operations → [LOGARITHM MACHINE] → Simple Operations
      |                          |                     |
      ▼                          ▼                     ▼
Multiplication    →    Addition
Division          →    Subtraction
Exponentiation    →    Multiplication
Root Extraction   →    Division
```

The Magic of Logarithmic Simplification:

Operation	Direct Calculation	Using Logarithms	Efficiency Gain
Multiplication	$32.5 \times 7.2 = ?$	$\log(32.5) + \log(7.2)$	Addition vs. complex multiplication
Division	$145.6 \div 8.3 = ?$	$\log(145.6) - \log(8.3)$	Subtraction vs. long division
Power	$(2.8)^5 = ?$	$5 \times \log(2.8)$	Multiplication vs. repeated multiplication

Operation	Direct Calculation	Using Logarithms	Efficiency Gain
Root	$\sqrt[3]{125} = ?$	$(1/3) \times \log(125)$	Division vs. trial and error

Real-World Historical Context

Before Calculators (1600s-1970s):

- **Scientists:** Used log tables for astronomical calculations
- **Engineers:** Designed bridges and buildings using logarithmic slide rules
- **Accountants:** Computed compound interest and amortization schedules

Visual: The Logarithmic Slide Rule

```
[Slide Rule Diagram]
│   1   2   3   4   5   6   7   8   9   1   │ ← C scale
|   |   |   |   |   |   |   |   |   |   |   |
│   1   2   3   4   5   6   7   8   9   1   │ ← D scale
```

Align scales to multiply: position 2 above 3, read 6 below

2. CONTENT

A. THE FOUR LAWS OF LOGARITHMIC CALCULATIONS

Law 1: Multiplication Law

$$\log(\ \times\ ) = \log + \log$$

Visual Proof:

If $M = a^x$ and $N = a^y$, then:

$M \times N = a^x \times a^y = a^{x+y}$

Therefore: $\log_a(M \times N) = x + y = \log_a M + \log_a N$

Law 2: Division Law

$$\log \left(—\right) = \log \quad - \log$$

Visual Proof:

If $M = a^x$ and $N = a^y$, then:

$M/N = a^x / a^y = a^{x-y}$

Therefore: $\log_a(M/N) = x - y = \log_a M - \log_a N$

Law 3: Power Law

$$\log() = \times \log$$

Visual Proof:

If $M = a^x$, then:

$M^p = (a^x)^p = a^{px}$

Therefore: $\log_a(M^p) = px = p \times \log_a M$

Law 4: Root Law

$$\log \quad \sqrt{()} = \frac{1}{} \times \log$$

Visual Proof:

If $M = a^x$, then:

$\sqrt[n]{M} = M^{1/n} = (a^x)^{1/n} = a^{x/n}$

Therefore: $\log_a(\sqrt[n]{M}) = x/n = (1/n) \times \log_a M$

Memory Aid: "MAD ROOT"

- **M**ultiplication → **A**ddition
- **D**ivision → **S**ubtraction
- **ROOT** → Division of log

B. STEP-BY-STEP PROCEDURE FOR LOGARITHMIC CALCULATIONS

Flowchart: The Logarithmic Calculation Process

```
Start with complex expression
        ↓
Break into components
        ↓
Create log table for each number
        ↓
Apply logarithmic laws
        ↓
Perform simple arithmetic on logs
        ↓
Use antilog table to convert back
        ↓
Final answer!
```

Step 1: Expression Analysis Identify all components and operations in your expression.

Step 2: Logarithm Table Setup Create a systematic working table:

Number	Logarithm	Notes
[Value 1]	[Lookup from table]	[Any special handling]
[Value 2]	[Lookup from table]	[Any special handling]

Step 3: Law Application Apply the appropriate logarithmic laws based on operations.

Step 4: Final Computation Combine the logarithmic results and use antilog tables.

C. MASTERING BAR NOTATION (NEGATIVE CHARACTERISTICS)

Understanding Bar Notation:

1.3̄979 = -1 + 0.3979 = -0.6021

2.8̄543 = -2 + 0.8543 = -1.1457

Visual: The Bar Notation Number Line

```
-2     -1      0      1      2
|----------|----------|----------|----------|
2.x̄xx 1.x̄xx  0.xxx  1.xxx  2.xxx
```

Operation Rules with Bar Logarithms:

1. Addition of Bar Logs:

2.3̄ + 1.5 = ?

= (-2 + 0.3) + (-1 + 0.5)

= -3 + 0.8

= 3.8̄

2. Subtraction of Bar Logs:

2.3̄ - 1.5̄ = ?

= (-2 + 0.3) - (-1 + 0.5)

= -2 + 0.3 + 1 - 0.5

= -1 - 0.2

= -1.2 = 2.8̄ (since -1.2 = -2 + 0.8)

3. Multiplication of Bar Logs:

2.7̄ × 3 = ?

= (-2 + 0.7) × 3

= -6 + 2.1

= -4 + 0.1 (since -6 + 2.1 = -3.9 = -4 + 0.1)

= 4.1̄

4. Division of Bar Logs:

```
3.4̄ ÷ 2 = ?
3.4̄ = -3 + 0.4 = -4 + 1.4  (borrow 1 from characteristic)
= ( -4 + 1.4 ) ÷ 2
= -2 + 0.7
= 2.7
```

Golden Rule: Never let the mantissa become negative! Always borrow from the characteristic to keep the mantissa positive.

3. DETAILED WORKED EXAMPLES

Example 6.1: Multiplication and Division

Evaluate: $\frac{38.4 \times 8.6}{15.2}$

Step-by-Step Solution:

Step 1: Logarithmic Transformation

$$\log\left(\frac{38.4 \times 8.6}{15.2}\right) = \log(38.4) + \log(8.6) - \log(15.2)$$

Step 2: Table Lookup and Setup

Number	Logarithm	Calculation
38.4	1.5843	From log table (384)
8.6	0.9345	From log table (860)
15.2	1.1818	From log table (152)

Step 3: Apply Logarithmic Laws

```
Numerator: 1.5843 + 0.9345 = 2.5188
Complete: 2.5188 - 1.1818 = 1.3370
```

Step 4: Antilog Conversion

```
Final log = 1.3370
Characteristic = 1, Mantissa = 0.3370
From antilog table: 0.3370 → 2173
Result = 2.173 × 10¹ = 21.73
```

Verification: $\frac{38.4 \times 8.6}{15.2} = \frac{330.24}{15.2} = 21.73$ ✓

✅ **Answer: 21.73**

Example 6.2: Powers and Roots

Evaluate: $\sqrt[3]{\frac{1.65^2}{29.4}}$

Step-by-Step Solution:

Step 1: Logarithmic Transformation

$$\log\left(\sqrt[3]{\frac{1.65^2}{29.4}}\right) = \frac{1}{3} \times [2 \times \log(1.65) - \log(29.4)]$$

Step 2: Table Lookup and Setup

Number	Logarithm	Calculation
1.65	0.2175	From log table
1.65^2	0.4350	2 × 0.2175
29.4	1.4683	From log table

Step 3: Apply Logarithmic Laws

```
Numerator: 2 × log(1.65) = 0.4350
Difference: log(1.65²) - log(29.4) = 0.4350 - 1.4683
```

Step 4: Handle Negative Result

```
0.4350 - 1.4683 = -1.0333
Express with bar notation: 1.9667
(Since -1.0333 = -2 + 0.9667)
```

Step 5: Apply Cube Root

```
1.9667 ÷ 3 = ?
1.9667 = -1 + 0.9667 = -2 + 1.9667 (borrow 1)
Now divide: (-2 + 1.9667) ÷ 3 = -0.6667 + 0.6556 = 1.6556
```

Step 6: Antilog Conversion

```
Final log = 1.6556
Mantissa 0.6556 → 4525 (from antilog table)
Characteristic -1 → 4.525 × 10⁻¹ = 0.4525
```

✅ **Answer: 0.4525**

Example 6.3: Real-World Application - Compound Interest

Problem: How many years will it take ₦50,000 to grow to ₦70,000 at 8% per year?

Compound Interest Formula:

$$= (1 +)$$

Where:

- A = Final amount (₦70,000)
- P = Principal (₦50,000)
- r = Interest rate (8% = 0.08)
- n = Number of years

Step-by-Step Solution:

Step 1: Set Up Equation

$$70{,}000 = 50{,}000(1.08)$$

$$1.4 = (1.08)$$

Step 2: Apply Logarithms

$$\log(1.4) = \times \log(1.08)$$

$$= \frac{\log(1.4)}{\log(1.08)}$$

Step 3: Table Lookup

Number	Logarithm
1.4	0.1461
1.08	0.0334

Step 4: Calculate

$$= \frac{0.1461}{0.0334} = 4.374$$

Step 5: Interpret Result

$$\approx 4.37 \text{ years} \approx 4 \text{ years and } 4.5 \text{ months}$$

✅ **Answer: 4.4 years**

4. TROUBLESHOOTING AND COMMON PITFALLS

Pitfall 1: Negative Mantissa

Incorrect: $\bar{2}.3 - \bar{1}.8 = \bar{1}.5$ (wrong!) **Correct:** $\bar{2}.3 - \bar{1}.8 = \bar{2}.5$ (borrow to keep mantissa positive)

Pitfall 2: Decimal Point Placement

Rule: Characteristic determines decimal placement:

- Positive characteristic: Decimal after (characteristic + 1) digits from left
- Negative characteristic: Decimal before first non-zero digit

Pitfall 3: Bar Notation Arithmetic

Always convert: $\bar{2}.7 = -2 + 0.7 = -1.3$ **Never write:** $\bar{2}.7 \neq -2.7$

5. REVISION QUESTIONS

1. How do logarithms transform multiplication into addition? Provide a mathematical proof.
2. Explain the process of handling division with bar notation logarithms.
3. Why must we never let the mantissa become negative during calculations?

6. ASSESSMENT

EVALUATION (IN-CLASS)

1. State the logarithmic law for:
   - (a) $\log(\times)$
   - (b) $\log()$
2. Set up the log table to calculate $= 5.2 \times 9.8$
3. Set up the log table to calculate $= \sqrt{5.2}$

ASSIGNMENT (TAKE-HOME)

Use 4-figure logarithm tables for all calculations.

1. Evaluate $\frac{403.2 \times 0.056}{10.8}$

2. Evaluate $\sqrt{45.6}$
3. Evaluate $(1.34)^5$

7. HISTORICAL PERSPECTIVE AND MODERN APPLICATIONS

The Invention of Logarithms

- **1614:** John Napier publishes "Mirifici Logarithmorum Canonis Descriptio"
- **1620:** Edmund Gunter creates the logarithmic scale
- **1632:** William Oughtred invents the slide rule

Modern Applications

- **Computer Science:** Logarithmic time complexity (O(log n))
- **Finance:** Compound interest and annuities
- **Science:** pH scale, Richter scale, decibel measurements
- **Engineering:** Signal processing and control systems

Memory Poem:

```
When numbers multiply, and you want to cry,
Use logs to add, and reach for the sky!
Division's a pain, but logs make it sane,
Just subtract them out, and knowledge you'll gain!

Powers and roots, once complex and deep,
Become simple arithmetic, secrets to keep.
With tables in hand, you're a math wizard grand,
The master of numbers throughout the land!
```

Answers to Assessment

In-Class:

1. (a) $\log(\times) = \log + \log$, (b) $\log() = \times \log$
2. $\log = \log 5.2 + \log 9.8$
3. $\log = \frac{1}{2} \times \log 5.2$

WEEK 7: SETS (I)

TOPIC: INTRODUCTION TO SETS

INTRODUCTION

In mathematics, **Sets** are the *foundation* of all topics — from algebra and geometry to logic and probability.

Think of a **set** as a *container* or *collection* that holds things (called **elements**) which share a common property.

Illustration 1 — Concept of a Set

```
Set of Fruits (F)
F = {, , , }
Each fruit is a distinct element of the set F.
```

Sets are used everywhere:

- The set of students in a class.
- The set of natural numbers.
- The set of letters in the English alphabet.

---

1. DEFINITION OF A SET

A **set** is a *well-defined collection of distinct objects* (called elements).
Keywords:

- **Well-defined:** You can clearly say if something belongs or doesn't belong to the set.
- **Distinct:** No repetition of elements.

✅ **Example (Well-defined)**: A = {The months of the year} → {January, February, …, December}

❌ **Not Well-defined:** B = {Beautiful flowers} → The word *beautiful* is subjective.

SET NOTATION

There are two main ways to describe sets:

A. Roster (Listing) Method

Elements are listed one by one, enclosed in curly brackets **{ }**.

Example: A = {2, 4, 6, 8, 10}

B. Set-builder Notation

We describe the property that defines the elements.

Example: $A = \{x \mid x \text{ is an even number}, 0 < x \leq 10\}$ (Read as: *A is the set of all x such that x is an even number between 0 and 10.*)

COMMON SYMBOLS IN SET THEORY

Symbol	Meaning	Example
$\in$	"is an element of"	$4 \in \{1,2,3,4,5\}$
$\notin$	"is not an element of"	$7 \notin \{1,2,3,4,5\}$
$\subset$	Subset	$A \subset B$
$\subseteq$	Subset (may be equal)	$A \subseteq A$
$\varnothing$	Empty Set	$\varnothing \subset A$
U	Universal Set	All elements considered

2. TYPES OF SETS

Type	Description	Example
Null Set ($\varnothing$)	Contains no elements	$\varnothing$ = { } (humans who can fly unaided)
Singleton Set	Has only one element	{The President of Nigeria}
Finite Set	Countable number of elements	{1, 2, 3, 4, 5}
Infinite Set	Endless elements	{1, 2, 3, 4, …}
Equal Sets	Contain exactly the same elements	{1,2,3} = {3,1,2}
Equivalent Sets	Contain same number of elements	{a,b,c} and {1,2,3}

Illustration 2 — Types of Sets

```
[ ∅ ] → Null Set
[ {A} ] → Singleton Set
[ {1,2,3} ] → Finite Set
[ {1,2,3,4,…} ] → Infinite Set
```

3. UNIVERSAL SET (U)

The **Universal Set (U)** is the set of all objects under discussion.
Example: If U = {1,2,3,4,5,6,7,8,9,10}, and A = {2,4,6,8,10}, then A is a subset of U.
Note: The Universal Set depends on context. For geometry, U may be "all triangles"; for probability, "all possible outcomes".

4. SUBSETS (⊂, ⊆)

If every element of A is also in B, then A is a subset of B.

Example: A = {1,2} B = {1,2,3,4} ⇒ A ⊂ B

Every set has:

- **Proper subsets:** Excludes the full set itself.
- **Improper subsets:** Includes the set itself and ∅ .

5. POWER SET (P(A))

The **power set** of a set A is the *set of all possible subsets of A.*

If n(A) = number of elements in A, then **n(P(A)) = 2^n.**

Example: A = {a, b} Subsets = ∅ , {a}, {b}, {a,b} Hence, P(A) = {∅ , {a}, {b}, {a,b}} and n(P(A)) = 4.

Illustration 3 — Power Set Tree

```
       {a,b}
     /       \
  {a}         {b}
    \          /
       ∅
```

WORKED EXAMPLES

Example 1: Let V = {a, e, i, o, u} a) Write in set-builder form. b) Is it finite or infinite? c) Find n(V). d) Is a ∈ V? e) Is b ∈ V?

✅ **Solution:** a) V = {x | x is a vowel in English alphabet} b) Finite (since we can count all 5 vowels) c) n(V) = 5 d) Yes, a ∈ V e) No, b ∉ V

Example 2: Let C = {red, green} a) List all subsets. b) Write the power set P(C). c) Find n(P(C)).

✅ **Solution:** Subsets = ∅ , {red}, {green}, {red, green} P(C) = {∅ , {red}, {green}, {red, green}} n(P(C)) = 4

REVISION QUESTIONS

1. Which of the following are well-defined? a) The set of interesting subjects in SS1. b) The set of rivers in Nigeria. c) The set of tasty meals.
2. List all elements of P = {x | x is a prime number and 10 < x < 25}.
3. Let A = {1, {2,3}, 4}. Is {2,3} ∈ A? Is 2 ∈ A?
4. If n(X) = 5, find the number of subsets and proper subsets of X.

ASSIGNMENT (TAKE-HOME)

1. Classify as finite or infinite: a) All leaves on trees in Lagos. b) A = {x | x is an integer < 0}. c) B = {people who are 300 years old}.
2. Let U = {x | 0 < x ⩽ 12, x ∈ N}, A = {factors of 12}. a) List elements of U. b) List elements of A. c) Is A ⊂ U?
3. If n(P) = k, find n(P(P)).
4. A set X has 63 proper subsets. Find the number of elements in X.
5. Given S = {a,b,c,d,e}, list all subsets with exactly 3 elements.

WEEK 8: SETS (II)

TOPIC: SET OPERATIONS AND VENN DIAGRAMS

INTRODUCTION

Last week, we described single sets. Now, we'll learn how **sets interact** — combining, overlapping, and excluding elements. The visual tool for this is the **Venn Diagram**, invented by **John Venn (1880)**.

Illustration 4 — Basic 2-Set Venn Diagram

```
┌─────────── U ──────┐
│      ○ A       ○ B       │
│   (Overlap = A ∩ B)      │
└────────────────────┘
```

1. SET OPERATIONS

Operation	Symbol	Definition	Example
Union	$\cup$	Elements in A or B or both	$A=\{1,2,3\}$, $B=\{3,4,5\}$, $A\cup B=\{1,2,3,4,5\}$
Intersection	$\cap$	Elements common to both	$A\cap B=\{3\}$
Complement	A'	Elements in U but not in A	If $U=\{1–5\}$, $A=\{1,2,3\}$, $A'=\{4,5\}$
Difference	$A\setminus B$	Elements in A but not in B	$A\setminus B=\{1,2\}$

Illustration 5 — Union and Intersection

```
Union (A∪B): Both circles shaded
Intersection (A∩B): Only the overlap shaded
```

2. PRINCIPLE OF INCLUSION–EXCLUSION

For two sets:

$$n(A\cup B) = n(A) + n(B) - n(A\cap B)$$

For three sets:

$$n(A\cup B\cup C) = n(A) + n(B) + n(C) - n(A\cap B) - n(A\cap C) - n(B\cap C) + n(A\cap B\cap C)$$

3. VENN DIAGRAMS

A **Venn Diagram** shows sets pictorially.

- The rectangle = Universal Set (U).
- Circles = Subsets (A, B, C).
- Overlaps = Intersections.

Illustration 6 — 3-Set Venn Diagram Layout

```
A ∩ B ∩ C (center)
A ∩ B only
A only
B only
C only
Outside region = (A∪B∪C)’
```

4. WORKED EXAMPLES

Example 1 — 2-Set Problem

In a class of 35 students, 20 offer Physics (P) and 17 offer History (H). 5 offer neither. Find: a) Students who offer both. b) Students who offer History only.

✅ **Solution:** $n(U)=35$, $n(P)=20$, $n(H)=17$, "Neither"=5

$n(P\cup H)=35-5=30$ Use inclusion-exclusion: $n(P\cup H)=n(P)+n(H)-n(P\cap H)$ $30=20+17-n(P\cap H)$
~ $n(P\cap H)=7$

a) **Both = 7** b) **History only = $n(H)-n(P\cap H)=17-7=10$**

Illustration 7 — 2-Set Diagram

```
   (A∩B=7)
A=20 total → A only=13
B=17 total → B only=10
Outside=5
```

Example 2 — 3-Set Problem

Data	Value
n(U)	100
n(F)	40
n(C)	30
n(P)	35

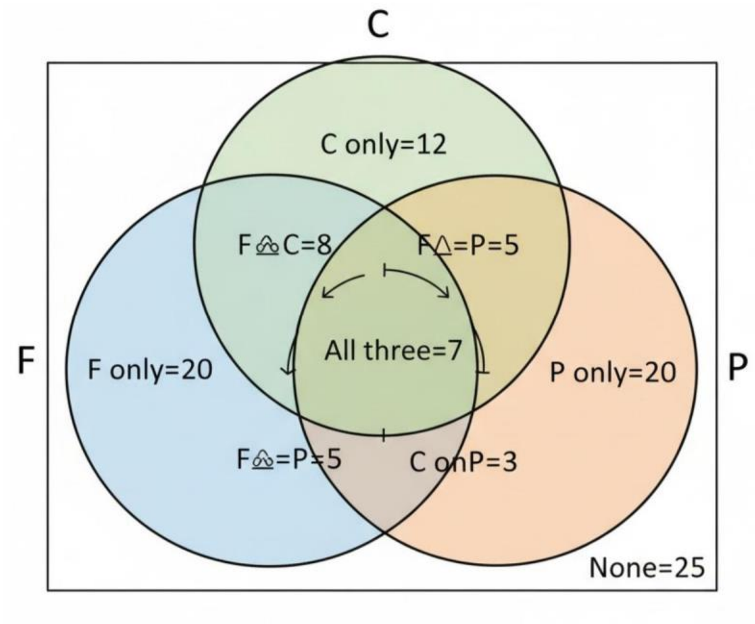
Data	Value
n(F∩C)	15
n(F∩P)	12
n(C∩P)	10
n(F∩C∩P)	7

✅***Solution:***

- $n(F\cap C \text{ only})=15-7=8$
- $n(F\cap P \text{ only})=12-7=5$
- $n(C\cap P \text{ only})=10-7=3$
- $n(F \text{ only})=40-(8+5+7)=20$
- $n(C \text{ only})=30-(8+3+7)=12$
- $n(P \text{ only})=35-(5+3+7)=20$

Total inside = 75 "None" = 100−75=25

Illustration 8 — 3-Set Venn Diagram



5. DE MORGAN’S LAWS

For any sets A and B:

1. $(A \cup B)' = A' \cap B'$
2. $(A \cap B)' = A' \cup B'$

Illustration: Overlapping circles showing how complements flip union/intersection.

REVISION QUESTIONS

1. If U={1,2,…,10}, A={1,3,5,7,9}, B={2,3,5,7}, find: a) $A \cup B$ b) $A \cap B$ c) A' d) $B \setminus A$
2. Shade region $B \cap A'$ in a Venn diagram.
3. State De Morgan's Laws.
4. If 15 play Football, 12 play Basketball, and 5 both, how many students are there if all play at least one?

ASSIGNMENT (TAKE-HOME)

1. Given U={a,b,c,d,e,f,g,h}, X={a,b,d,e}, Y={b,e,f,g}: Find: a) $X' \cap Y$ b) $(X \cup Y)'$ c) Verify $(X \cup Y)' = X' \cap Y'$
2. In a group of 50 people, 30 speak English, 25 speak French, and 10 speak neither: a) Draw a Venn diagram. b) Find number who speak both. c) Find English only.
3. A survey of 60 fans: 28 watch Football (F), 22 Basketball (B), 25 Cricket (C), 10 $F \cap B$, 8 $F \cap C$, 7 $B \cap C$, 3 all three. a) Fill all 8 regions. b) How many watch exactly two? c) How many watch none?
4. If $n(A \setminus B)=15$, $n(B \setminus A)=10$, $n(A \cap B)=5$, find $n(A \cup B)$.
5. Draw and shade region $A \cap (B \cup C)'$.

WEEK 9: SIMPLE EQUATIONS AND VARIATIONS

Concept: Mastering Algebraic Manipulation and Proportional Relationships

1. INTRODUCTION: THE LANGUAGE OF MATHEMATICS

Algebraic manipulation is the foundation of all advanced mathematics and scientific reasoning. This week, we unlock two powerful tools that appear throughout STEM fields.

Real-World Applications

Diagram 1: Formulas in Everyday Life

SCIENCE & ENGINEERING	FINANCE & ECONOMICS
↓	↓
$F = ma$ (Physics)	$A = P(1 + r)^n$ (Compound Interest)
$v = u + at$ (Motion)	$I = Prt$ (Simple Interest)
$PV = nRT$ (Chemistry)	$C = \pi d$ (Circumference)
GEOMETRY & MEASUREMENT	COMPUTER SCIENCE
↓	↓
$A = \pi r^2$ (Circle area)	$y = mx + c$ (Linear algorithms)
$V = \frac{1}{3}\pi r^2 h$ (Cone volume)	$d = \sqrt{[(x_2-x_1)^2+(y_2-y_1)^2]}$ (Distance)

The Power of Rearrangement: Being able to manipulate formulas means you can:

- Calculate any variable when others are known
- Solve real-world problems efficiently
- Understand relationships between quantities
- Prepare for advanced mathematics and sciences

2. CONTENT

A. CHANGE OF SUBJECT OF FORMULA

Definition: The **subject** of a formula is the variable that stands alone on one side of the equation (typically the left side).

Visual: Identifying the Subject

```
Original formula: A = πr²
                  │   │
                  │   └─ Expression containing other variables
                  └───── SUBJECT (stands alone)
```

Changing the subject means rearranging the formula to make a different variable the subject.

The Golden Rule: Balance Principle

Visual: The Equation as a Balance Scale

```
     Left Side     =     Right Side

    ┌──────────┐        ┌──────────┐
    │  3x + 2  │        │    14    │
    └──────────┘        └──────────┘

         ║                    ║
         ║                    ║
    Must remain balanced!


Operation: Subtract 2 from both sides

    ┌──────────┐        ┌──────────┐
    │    3x    │        │    12    │
    └──────────┘        └──────────┘


Operation: Divide both sides by 3

    ┌──────────┐        ┌──────────┐
```

```
│    x    │          │    4    │
└─────────┘          └─────────┘
```

Key Principle: Whatever operation you perform on one side, you MUST perform on the other side to maintain balance.

Reverse BODMAS Strategy

Diagram 2: The "Unwrapping" Analogy

```
Original: 2(x + 3) = 14
          │ │   │    │
          │ │   │    └─ Outer layer
          │ │   └────── Middle layer
          │ └────────── Inner layer
          └──────────── Innermost layer

UNWRAPPING ORDER (Reverse BODMAS):
1. Remove outer layer (÷2): (x + 3) = 7
2. Remove middle layer (-3): x = 4
3. Innermost layer revealed: x = 4
```

Systematic Approach:

Step	Operation to Undo	Example	Action
1	**B**rackets	$2(x + 3) = 14$	Expand or factor
2	**O**rders (powers/roots)	$x^2 = 16$	Square root both sides
3	**D**ivision/Multiplication	$3x = 12$	Divide by 3
4	**A**ddition/Subtraction	$x + 5 = 9$	Subtract 5
5	**S**ubject isolated	$x = 4$	Solution found

Operation Inverses Reference Table

Operation	Inverse Operation	Visual Example
+ a	- a	x + 5 = 8 → x = 8 - 5
- a	+ a	x - 3 = 7 → x = 7 + 3
× a	÷ a	4x = 20 → x = 20 ÷ 4
÷ a	× a	x/2 = 6 → x = 6 × 2
x^2	√x	$x^2 = 25 \rightarrow x = \sqrt{25}$
√x	x^2	$\sqrt{x} = 3 \rightarrow x = 3^2$
$^1/_x$	Reciprocal	1/x = 2 → x = 1/2

DETAILED WORKED EXAMPLES

Example 8.1: Handling Brackets

Make x the subject of: $(+b) =$ **Step-**

by-Step Solution:

```
Step 1: Divide both sides by a (undo multiplication)
a(x + b)     c
───────── = ───
   a         a
x + b = c/a

Step 2: Subtract b from both sides (undo addition)
x + b - b = c/a - b
x = c/a - b

Step 3: Common denominator (optional simplification)
x = (c - ab)/a
```

Visual Process:

```
a(x + b) = c
   ↓ [÷a]
x + b = c/a
   ↓ [-b]
x = c/a - b
```

✓ **Answer:** $= {}^{-b}$ ___

Example 8.2: Dealing with Roots

Make L the subject of: $= 2 \quad \sqrt{-}$

Step-by-Step Solution:

```
Step 1: Divide by 2π (undo multiplication)
T/(2π) = √(L/g)

Step 2: Square both sides (undo square root)
[T/(2π)]² = L/g
T²/(4π²) = L/g

Step 3: Multiply by g (undo division)
L = gT²/(4π²)
```

Visual Process:

```
T = 2π√(L/g)
 ↓ [÷2π]
T/(2π) = √(L/g)
 ↓ [²]
T²/(4π²) = L/g
 ↓ [×g]
L = gT²/(4π²)
```

✓ **Answer:** $= \frac{{}^{2}}{4^{2}}$

Real-World Connection: This is the formula for the period of a pendulum!

Example 8.3: Variable on Both Sides

Make x the subject of: $= \frac{+1}{-2}$

Step-by-Step Solution:

```
Step 1: Multiply both sides by (x - 2)
y(x - 2) = x + 1

Step 2: Expand the left side
xy - 2y = x + 1

Step 3: Collect x terms on one side
xy - x = 1 + 2y

Step 4: Factor out x
x(y - 1) = 1 + 2y

Step 5: Divide by (y - 1)
x = (1 + 2y)/(y - 1)
```

Visual Process:

```
    x + 1
y = ─────
    x - 2
 ↓ [×(x-2)]
y(x - 2) = x + 1
 ↓ [Expand]
xy - 2y = x + 1
```

```
 ↓ [Rearrange]
xy - x = 1 + 2y
 ↓ [Factor]
x(y - 1) = 1 + 2y
 ↓ [÷(y-1)]
x = (1 + 2y)/(y - 1)
```

✅ **Answer:** $= \frac{1+2}{-1}$

Example 8.4: Powers and Denominators

Make x the subject of: $= \frac{3^2+1}{2}$

Step-by-Step Solution:

```
Step 1: Multiply both sides by 2
2y = 3x² + 1

Step 2: Subtract 1
2y - 1 = 3x²

Step 3: Divide by 3
(2y - 1)/3 = x²

Step 4: Take square root (remember ±)
x = ±√[(2y - 1)/3]
```

Important Note: When taking square roots, we include ± because both positive and negative values satisfy the equation.

✅ **Answer:** $=\pm \sqrt{\frac{2-1}{3}}$

B. SUBSTITUTION INTO FORMULAS

Definition: Substitution means replacing variables with given numerical values to calculate a result.

Process:

1. Write the formula
2. Replace each variable with its given value
3. Follow BODMAS rules to compute
4. Include appropriate units

Example 8.5: Volume of a Cylinder

Given: $= {}^{2}h$, find V when $= \frac{22}{7}$, r = 7, h = 5

Step-by-Step Solution:

```
V = πr²h
  = (22/7) × (7)² × 5
  = (22/7) × 49 × 5
  = 22 × 7 × 5
  = 22 × 35
  = 770
```

Calculation Check:

```
(22/7) × 49 = 22 × 7 = 154
154 × 5 = 770
```

✅ **Answer:** V = 770

Example 8.6: Compound Interest

Given: $= (1 + t)$, find A when P = 2000, r = 0.05, t = 3

Step-by-Step Solution:

```
A = P(1 + rt)
  = 2000 × (1 + 0.05 × 3)
  = 2000 × (1 + 0.15)
  = 2000 × 1.15
  = 2300
```

Financial Interpretation: ₦2,000 invested at 5% simple interest for 3 years grows to ₦2,300

✅ **Answer:** A = 2300

C. VARIATION: MATHEMATICAL RELATIONSHIPS

Concept: Variation describes how one quantity changes in relation to another.

Diagram 3: The Four Types of Variation

```
DIRECT VARIATION        INVERSE VARIATION
    y                       y
    |                       |
    |   /                   |    \
    |  /                    |     \
    | /                     |      \
    |/                      |       \
    └───── x               └─────── x
    "More x → More y"       "More x → Less y"

JOINT VARIATION          COMBINED VARIATION
y varies with both        Mix of direct and
x and z together          inverse relationships
```

Variation Types Reference Table

Type	Statement	Symbolic Form	Formula	Real-World Example
Direct	"y varies directly as x"	$y \propto x$	$y = kx$	Distance vs. Time (constant speed)
Inverse	"y varies inversely as x"	$y \propto 1/x$	$y = k/x$	Pressure vs. Volume (Boyle's Law)
Joint	"y varies jointly as x and z"	$y \propto xz$	$y = kxz$	Area of rectangle ($A = l \times w$)
Combined	Mixed direct and inverse	$y \propto x^2/\sqrt{z}$	$y = kx^2/\sqrt{z}$	Gravitational force

Solving Variation Problems: 5-Step Method

Step 1: Write the proportional statement **Step 2:** Convert to equation using constant k **Step 3:** Use given data to find k **Step 4:** Write the complete formula **Step 5:** Solve for unknown values

DETAILED VARIATION EXAMPLES

Example 8.7: Direct Variation

Problem: y varies directly as the square of x. When $x = 3$, $y = 18$. Find y when $x = 5$.

Step-by-Step Solution:

```
Step 1: Write proportional statement
y ∝ x²


Step 2: Convert to equation
y = kx²


Step 3: Find k using given data
```

```
18 = k × (3)²
18 = 9k
k = 2

Step 4: Write complete formula
y = 2x²

Step 5: Solve for unknown
When x = 5: y = 2 × (5)² = 2 × 25 = 50
```

Graphical Insight: This is a parabola opening upward

✅ **Answer:** $y = 50$

Example 8.8: Inverse Variation

Problem: R varies inversely as $\sqrt{V}$. When $R = 10$, $V = 25$, find V when $R = 5$.

Step-by-Step Solution:

```
Step 1: Write proportional statement
R ∝ 1/√V

Step 2: Convert to equation
R = k/√V

Step 3: Find k using given data
10 = k/√25
10 = k/5
k = 50

Step 4: Write complete formula
R = 50/√V
```

```
Step 5: Solve for unknown
5 = 50/√V
√V = 50/5 = 10
V = 10² = 100
```

Real-World Connection: Electrical resistance vs. conductor diameter

✅ **Answer:** V = 100

Example 8.9: Joint Variation

Problem: y varies jointly as x and z. When x = 2, z = 3, y = 24, find y when x = 4, z = 6.

Step-by-Step Solution:

```
Step 1: Write proportional statement
y ∝ xz

Step 2: Convert to equation
y = kxz

Step 3: Find k using given data
24 = k × 2 × 3
24 = 6k
k = 4

Step 4: Write complete formula
y = 4xz

Step 5: Solve for unknown
When x = 4, z = 6: y = 4 × 4 × 6 = 96
```

Observation: Both x and z doubled, so y quadrupled (2 × 2 = 4 times)

✅ **Answer:** y = 96

Example 8.10: Combined Variation

Problem: y varies directly as x^2 and inversely as $\sqrt{z}$. When x = 4, z = 9, y = 8, find y when x = 6, z = 16.

Step-by-Step Solution:

```
Step 1: Write proportional statement
y ∝ x²/√z

Step 2: Convert to equation
y = kx²/√z

Step 3: Find k using given data
8 = k × (4)² / √9
8 = k × 16 / 3
8 = 16k/3
k = 8 × 3/16 = 24/16 = 1.5

Step 4: Write complete formula
y = 1.5x²/√z

Step 5: Solve for unknown
When x = 6, z = 16: y = 1.5 × (6)² / √16
y = 1.5 × 36 / 4 = 54 / 4 = 13.5
```

✅ **Answer:** y = 13.5

3. GRAPHICAL REPRESENTATION OF VARIATION

Diagram 4: Variation Graphs

```
DIRECT: y = kx        INVERSE: y = k/x
    y                       y
    |                       |
    |     /                 |    \
    |    /                  |     \
    |   /                   |      \
    |  /                    |       \
    | /                     |        \
    └──────  x              └─────────  x

DIRECT SQUARE: y = kx²   INVERSE SQUARE: y = k/x²
    y                       y
    |                       |
    |    /                  |\
    |   /                   | \
    |  /                    |  \
    | /                     |   \
    |/                      |    \
    └──────  x              └──────  x
```

4. REAL-WORLD APPLICATIONS TABLE

Situation	Variation Type	Formula	Explanation
Speed	Direct	$d = st$	Double time → Double distance
Boyle's Law	Inverse	$P \propto 1/V$	Half volume → Double pressure
Gravity	Inverse Square	$F \propto 1/r^2$	Double distance → ¼ force
Circle Area	Direct Square	$A \propto r^2$	Double radius → 4× area
Light Intensity	Inverse Square	$I \propto 1/d^2$	Double distance → ¼ brightness
Electrical Resistance	Inverse	$R \propto 1/A$	Double wire area → Half resistance

5. REVISION QUESTIONS

1. Define "subject of a formula" and explain why changing the subject is useful.
2. State the golden rule for equation manipulation and illustrate with an example.
3. Convert these statements to equations:
    - "y varies directly as the cube of x"
    - "P varies inversely as the square root of t"
    - "A varies jointly as b and h"
4. Explain the difference between direct and inverse variation using real-world examples.
5. If y doubles when x doubles, what type of variation is this? What if y halves when x doubles?

6. ASSESSMENT

EVALUATION (IN-CLASS)

1. Make C the subject of: $= \frac{9}{5} + 32$
2. Make r the subject of: $= (1 + t)$
3. If y ∝ x and y = 10 when x = 2, find y when x = 7
4. If y ∝ 1/x and y = 5 when x = 4, find y when x = 10
5. If P ∝ L^2 and L is doubled, what happens to P?

ASSIGNMENT (TAKE-HOME)

1. Make h the subject of: $= 2(+ h)$
2. Make x the subject of: $= \sqrt{\frac{}{b-}}$
3. If $= \frac{1}{3}{}^2h$, find V when $\pi = 22/7$, r = 3, h = 7
4. F varies inversely as d^2. When d = 2 cm, F = 5 N. Find F when d = 5 cm
5. Z varies directly as x^2 and inversely as y. When x = 2, y = 3, Z = 8. Find Z when x = 3, y = 2